5.12 CURRENT LOOP AS A MAGNETIC DIPOLE

14. *Show that a current carrying loop behaves as a magnetic dipole. Hence write an expression for its magnetic dipole moment.*

Current loop as a magnetic dipole. We know that the magnetic field produced at a large distance r from the centre of a circular loop (of radius a) along its axis is given by

$$B = \frac{\mu_0 Ia^2}{2r^3}$$

or
$$B = \frac{\mu_0}{4\pi} \cdot \frac{2IA}{r^3} \qquad ...(1)$$

where I is the current in the loop and $A = \pi a^2$ is its area. On the other hand, the electric field of an electric dipole at an axial point lying far away from it is given by

$$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{2p}{r^3} \qquad ...(2)$$

where p is the electric dipole moment of the electric dipole.

On comparing equations (1) and (2), we note that both B and E have same distance dependence $\left(\frac{1}{r^3}\right)$. Moreover, they have same direction at any far away point, not just on the axis. This suggests that a circular current loop behaves as a magnetic dipole of magnetic moment,

$$m = IA$$

In vector notation,

$$\vec{m} = I\vec{A} = IA\hat{n}$$

This result is valid for planar current loop of any shape. *Thus the magnetic dipole moment of any current loop is equal to the product of the current and its loop area.* Its direction is defined to be normal to the plane of the loop in the sense given by right hand thumb rule.

Right hand thumb rule. *If we curl the fingers of the right hand in the direction of current in the loop, then the extended thumb gives the direction of the magnetic moment associated with the loop.*

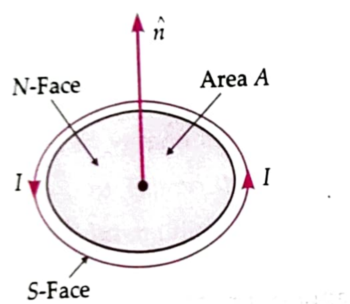

Fig. 5.20 Current loop as a magnetic dipole.

It follows from the above rule that the upper face of the current loop shown in Fig. 5.20. has N-polarity and the lower face has S-polarity. Thus *a current loop behaves like a magnetic dipole.*

If a current carrying coil consists of N turns, then

$$m = NIA$$

The factor NI is called *amperes turns* of current loop.

So,

Magnetic dipole moment of current loop

= Ampere turns × loop area

Clearly, dimensions of magnetic moment

$$= [A][L^2] = [AL^2]$$

SI unit of magnetic dipole moment is Am^2. *It is defined as the magnetic moment associated with one turn loop of area one square metre when a current of one ampere flows through it.*

Table 5.1 Analogy between electric and magnetic dipoles

Physical quantity	Electrostatics	Magnetism
Free space constant	$\frac{1}{\varepsilon_0}$	μ_0
Dipole moment	$\vec{p}$	$\vec{m}$
Axial field	$\frac{1}{4\pi\varepsilon_0} \cdot \frac{2\vec{p}}{r^3}$	$\frac{\mu_0}{4\pi} \cdot \frac{2\vec{m}}{r^3}$
Equatorial field	$-\frac{1}{4\pi\varepsilon_0} \cdot \frac{\vec{p}}{r^3}$	$-\frac{\mu_0}{4\pi} \cdot \frac{\vec{m}}{r^3}$
Torque in external field	$\vec{p} \times \vec{E}$	$\vec{m} \times \vec{B}$
P.E. *in external field*	$-\vec{p} \cdot \vec{E}$	$-\vec{m} \cdot \vec{B}$

5.13 MAGNETIC DIPOLE MOMENT OF A REVOLVING ELECTRON

15. *Derive an expression for the magnetic dipole moment of an electron revolving around a nucleus. Define Bohr magneton and find its value.*

Magnetic dipole moment of a revolving electron. According to Bohr model of hydrogen-like atoms, negatively charged electron revolves around the positively charged nucleus. This uniform circular motion of the electron is equivalent to a current loop which possesses a magnetic dipole moment = IA. As shown in Fig. 5.21, consider an electron revolving anticlockwise around a nucleus in an orbit of radius r with speed v and time period T.

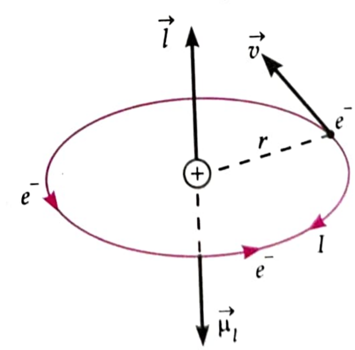

Fig. 5.21 Orbital magnetic moment of a revolving electron.

Equivalent current,

$$I = \frac{\text{Charge}}{\text{Time}} = \frac{e}{T} = \frac{e}{2\pi r / v} = \frac{ev}{2\pi r}$$

Area of the current loop, $A = \pi r^2$

Therefore, the *orbital magnetic moment* (magnetic moment due to orbital motion) of the electron is

$$\mu_l = IA = \frac{ev}{2\pi r} \cdot \pi r^2$$

or $$\mu_l = \frac{evr}{2} \quad ...(1)$$

As the negatively charged electron is revolving anticlockwise, the associated current flows clockwise. According to right hand thumb rule, the direction of the magnetic dipole moment of the revolving electron will be perpendicular to the plane of its orbit and in the downward direction, as shown in Fig. 5.21.

Also, the angular momentum of the electron due to its orbital motion is

$$l = m_e vr \quad ...(2)$$

The direction of $\vec{l}$ is normal to the plane of the electron orbit and in the upward direction, as shown in Fig. 5.21.

Dividing equation (1) by (2), we get

$$\frac{\mu_l}{l} = \frac{evr/2}{m_e vr} = \frac{e}{2m_e}$$

The above ratio is a constant called ***gyromagnetic ratio***. Its value is 8.8×10^{10} C kg^{-1}. So

$$\mu_l = \frac{e}{2m_e} l$$

Vectorially,

$$\vec{\mu}_l = -\frac{e}{2m_e} \vec{l}$$

The negative sign shows that the direction of $\vec{l}$ is opposite to that of $\vec{\mu}_l$. According to *Bohr's quantisation condition*, the angular momentum of an electron in any permissible orbit is integral multiple of $h/2\pi$, where h is Planck's constant, *i.e.*,

$$l = \frac{nh}{2\pi}, \quad \text{where } n = 1, 2, 3, ...$$

$$\therefore \quad \mu_l = n\left(\frac{eh}{4\pi m_e}\right)$$

This equation gives orbital magnetic moment of an electron revolving in nth orbit.

Bohr magneton. *It is defined as the magnetic moment associated with an electron due to its orbital motion in the first orbit of hydrogen atom.* It is the minimum value of μ_l which can be obtained by putting $n = 1$ in the above equation. Thus Bohr magneton is given by

$$\mu_B = (\mu_l)_{min} = \frac{eh}{4\pi m_e}$$

Putting the values of various constants, we get

$$\mu_B = \frac{1.6 \times 10^{-19} \text{ C} \times 6.63 \times 10^{-34} \text{ Js}}{4 \times 3.14 \times 9.11 \times 10^{-31} \text{ kg}}$$

$$= 9.27 \times 10^{-24} \text{ Am}^2.$$

Besides the orbital angular momentum $\vec{l}$, an electron has spin angular momentum $\vec{S}$ due to its spinning motion. *The magnetic moment possessed by an electron due to its spinning motion is called* ***intrinsic magnetic moment*** *or* ***spin magnetic moment***. It is given by

$$\vec{\mu}_s = -\frac{e}{m_e} \vec{S}$$

The total magnetic moment of the electron is the vector sum of these two momenta. It is given by

$$\vec{\mu} = \vec{\mu}_l + \vec{\mu}_s = -\frac{e}{2m_e}(\vec{l} + 2\vec{S})$$

Examples based on Torque and Potential Energy of a Dipole, and Magnetic Moment of a Current Loop

Formulae Used

1. Torque, $\tau = mB \sin\theta$ or $\vec{\tau} = \vec{m} \times \vec{B}$
2. Work done in turning the dipole or P.E. of a dipole,
   $W = U = -mB(\cos\theta_2 - \cos\theta_1)$
3. If initially the dipole is perpendicular to the field,
   $U = -mB\cos\theta$
   (*i*) When $\vec{m}$ is parallel to $\vec{B}$, $\theta = 0°$, $U = -mB$
   Potential energy of the dipole is minimum. It is in a state of stable equilibrium.
   (*ii*) When $\vec{m}$ is *perpendicular* to $\vec{B}$, $\theta = 90°$, $U = 0$.
   (*iii*) When $\vec{m}$ is *antiparallel* to $\vec{B}$, $\theta = 180°$, $U = +mB$
   Potential energy of the dipole is maximum. It is in a state of unstable equilibrium.
4. Magnetic moment of a current loop, $m = NIA$
5. Orbital magnetic moment of an electron in nth orbit,
   $$\mu_l = \frac{evr}{2} = \frac{e}{2m_e} l = n\left(\frac{eh}{4\pi m_e}\right)$$
6. Bohr magneton is the magnetic moment of an electron in first ($n = 1$) orbit.
   $$\mu_B = (\mu_l)_{min} = \frac{eh}{4\pi m_e}.$$

Units Used

Torque τ is in Nm, magnetic moment m in JT^{-1} or Am^2, field B in tesla, potential energy U in joule.

5.14 BAR MAGNET AS AN EQUIVALENT SOLENOID

16. ***State some similarities between a current carrying solenoid and a bar magnet.***

Similarities between a current carrying solenoid and a bar magnet. When a current is passed through a solenoid, it behaves like a bar magnet. Some observations of similar behaviour are as follows :

1. A current carrying solenoid suspended freely always comes to rest in north-south direction.
2. Two current-carrying solenoids exhibit mutual attraction and repulsion when brought closer to one another. This shows that their end faces act as N-and S-poles like that of a bar magnet.
3. Figure 5.22 shows the lines of force of a bar magnet while Fig. 5.23 shows the lines of force of a finite solenoid. The two patterns have a striking resemblance.

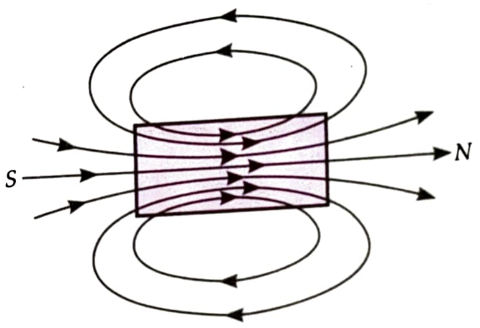

Fig. 5.22 **Field lines of a bar magnet.**

If we move a small compass needle in the neighbourhood of the bar magnet and the current carrying finite solenoid, we shall find that deflections of the needle are similar in the two cases. This again supports the similarity between the two fields.

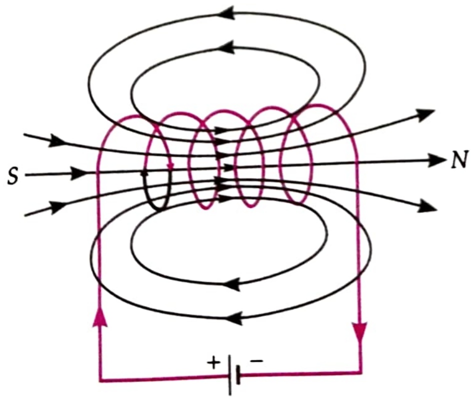

Fig. 5.23 **Field lines of a current carrying finite solenoid.**

4. The magnetic fields of both the bar magnet and current carrying solenoid at any far away axial point are given by the same expression :

$$B_{axial} = \frac{\mu_0}{4\pi} \cdot \frac{2m}{r^3}$$

Thus a bar magnet and a solenoid produce similar magnetic fields.

17. *Explain how is a current carrying solenoid equivalent to a bar magnet.*

A solenoid as an equivalent bar magnet. A solenoid can be regarded as a combination of circular loops placed side by side, as shown in Fig. 5.24(*a*). Each turn of the solenoid can be regarded as a small magnetic dipole of dipole moment IA. Then the solenoid becomes an arrangement of small magnetic dipoles placed in line with each other, as shown in Fig. 5.24(*b*). The number of such dipoles is equal to the number of turns in the solenoid. The north pole of one touches the south of the adjacent one. The opposite poles neutralise each other except at the ends. Thus, a current carrying solenoid can be replaced by just a single south pole and a single north pole, separated by a distance equal to the length of the solenoid. Hence a current carrying solenoid is equal to a bar magnet as shown in Fig. 5.24(*c*).

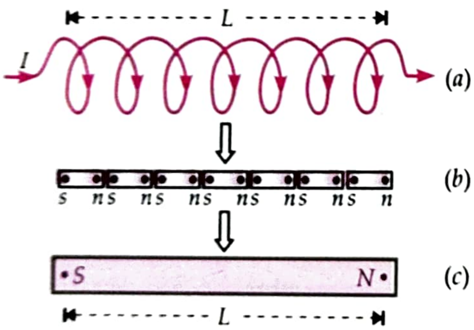

Fig. 5.24 **A solenoid as an equivalent bar magnet.**

A bar magnet and a finite solenoid produce similar magnetic field patterns, as shown in Fig. 5.22 and Fig. 5.23 respectively. It may be noted that the magnetic field inside the solenoid is in direction opposite to that we expect on the basis of the above pole model ($N \rightarrow S$).

To make this analogy more firm, we now calculate the axial field of a finite solenoid.

Expression for the magnetic field at an external point lying on its axis. As shown in Fig. 5.25, let the solenoid consist of n turns per unit length. Let its length be $2l$ and radius a. To evaluate its magnetic field at an axial point P at distance r from the centre O, we consider a circular element of thickness dx of the solenoid at distance x from its centre. It has ndx number of turns. Let I be current in the solenoid.

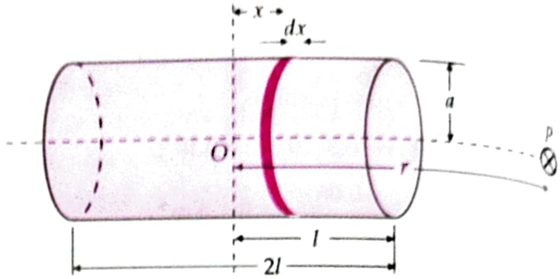

Fig. 5.25 **Calculation of axial field of a finite solenoid to show its similarity to a bar magnet.**

Magnetic field at point P due to the circular element,

$$dB = \frac{\mu_0 n dx I a^2}{2[(r-x)^2 + a^2]^{\frac{3}{2}}}$$

Total magnetic field at point P will be

$$B = \frac{\mu_0 n I a^2}{2} \int_{-l}^{+l} \frac{dx}{[(r-x)^2 + a^2]^{\frac{3}{2}}}$$

For far away axial point P, $r >> a$ and $r >> l$, so

$$B = \frac{\mu_0 n I a^2}{2r^3} \int_{-l}^{+l} dx = \frac{\mu_0 n I 2 l a^2}{2r^3}$$

$$B = \frac{\mu_0}{4\pi} \frac{2m}{r^3}$$

This expression is same as that the magnetic field of a bar magnet at far away axial point. Thus, a magnet and a solenoid produce similar magnetic fields. The magnetic moment of a bar magnet is equal to the magnetic moment of an equivalent solenoid that produces the same field.

18. *Explain how is a bar magnet equivalent to a current carrying solenoid.*

A bar magnet as an equivalent solenoid. We can explain this by ***Ampere's hypothesis*** according to which *all magnetic effects are produced by current-loops*. The electrons in an atom keep on revolving around its nucleus and hence set up electric currents. These atomic currents are equivalent to small circular current-loops. In a magnet, these current-loops are arranged parallel to each other and have currents in the same sense.

5.17 ORIGIN OF EARTH'S MAGNETIC FIELD

21. *Give a brief account of different theories regarding the source of earth's magnetism.*

Origin of earth's magnetic field. The magnetic field of the earth is approximately like that of a *giant bar magnet* embedded deep inside the earth. Many theories have been proposed about the cause of earth's magnetism from time to time. Some of these are mentioned below :

1. In 1600, *William Gilbert* in his book '*De Magnete*' first suggested that the earth behaves as a bar magnet and its magnetism is due to the presence of magnetic material at its centre, which could be a permanent magnet. However, the core of the earth is so hot that a permanent magnet cannot exist there.

2. *Prof. Blackett* suggested that the earth's magnetism is due to the rotation of the earth about its own axis. Every substance is made of charged particles such as protons and electrons. As these particles mo... along with the earth, they cause circulating curr... which, in turn, magnetise the earth.

3. Cosmic rays cause the ionisation of gases in the earth's atmosphere. As the earth rotates, strong electric currents are set up due to the movement of the charged ions. These currents may be the source of earth's magnetism.

4. According to *Sir E. Bullard* (U.K.) and W... *Elaster* (U.S.A.), there are large deposits of ferro-magnetic materials like iron, nickel, etc. in the core of the earth. The core of the earth is very hot and molten. *The circulating ions in the highly conducting liquid region of the earth's core form current loops and hence produce magnetic field.* At present, this hypothesis seems most probable because our moon, which has no molten core, has no magnetic field. Venus, which has a slower rate of rotation, has a weaker magnetic field while Jupiter with a faster rate of rotation has a stronger magnetic field.

The changes in the earth's magnetic field are so complicated and irregular that the exact cause of earth's magnetism is yet to be known.

5.18 SOME DEFINITIONS IN CONNECTION WITH EARTH'S MAGNETISM

22. *Define the terms geographic axis, magnetic axis, magnetic equator, magnetic meridian and geographic meridian in connection with geomagnetism.*

Some definitions in connection with earth's magnetism. Fig. 5.28 shows the magnetic lines of force around the earth.

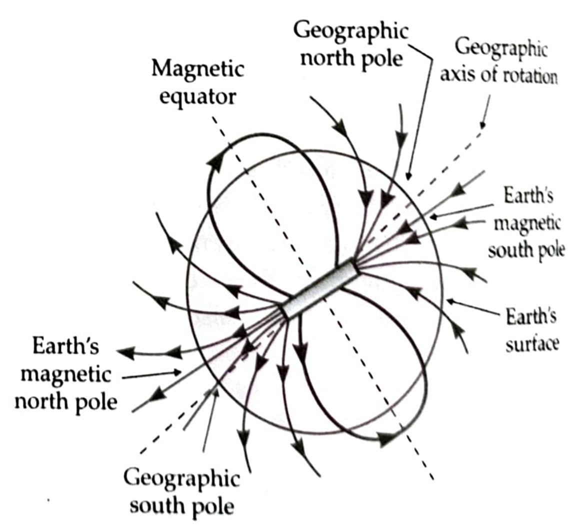

Fig. 5.28 Magnetic field of the earth.

1. Geographic axis. *The straight line passing through the geographical north and south poles of the earth is called its geographic axis. It is the axis of rotation of the earth.*

2. Magnetic axis. *The straight line passing through the magnetic north and south poles of the earth is called its magnetic axis.*

The magnetic axis of the earth makes an angle of nearly 20° with the geographic axis. At present, the magnetic south pole S_m is located at a point in Northern Canada at a latitude of 70.5°N and a longitude of 96°W. The magnetic north pole N_m is located diametrically opposite to S_m *i.e.*, at a latitude of 70.5°S and a longitude of 84°E. The magnetic poles are nearly 2000 km away from the geographic poles. The magnetic equator intersects the geographic equator at longitudes of 6°W and 174°E.

3. Magnetic equator. *It is the great circle on the earth perpendicular to the magnetic axis.*

4. Magnetic meridian. *The vertical plane passing through the magnetic axis of a freely suspended small magnet is called magnetic meridian. The earth's magnetic field acts in the direction of the magnetic meridian.*

5. Geographic meridian. *The vertical plane passing through the geographic north and south poles is called geographic meridian.*

5.19 ELEMENTS OF EARTH'S MAGNETIC FIELD

23. *What are the elements of earth's magnetic field ? Explain their meanings. Show these elements in a labelled diagram and deduce various relations between them.*

Elements of earth's magnetic field. *The earth's magnetic field at a place can be completely described by three parameters which are called elements of earth's magnetic field.* They are declination, dip and horizontal component of earth's magnetic field.

1. Magnetic declination. *The angle between the geographical meridian and the magnetic meridian at a place is called the magnetic declination (α) at that place.*

Magnetic declination arises because the magnetic axis of the earth does not coincide with its geographic axis.

To determine magnetic declination at a place, set up a compass needle that is free to rotate in a horizontal plane about a vertical axis, as shown in Fig. 5.29. The angle α that this needle makes with the geographic north-south ($N_g - S_g$) direction is the magnetic declination. By knowing declination, we can determine the vertical plane in which the earth's magnetic field lies. In India, the value of α is small. It is 0°41′ E for Delhi and 0°58′ W for Mumbai. This means that the N-pole of a compass needle almost points in the direction of geographic north.

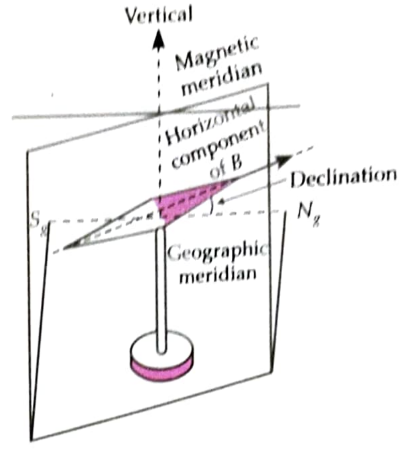

Fig. 5.29 Determination of declination at a place.

2. Angle of dip or magnetic inclination. *The angle made by the earth's total magnetic field $\vec{B}$ with the horizontal direction in the magnetic meridian is called angle of dip (δ) at any place.*

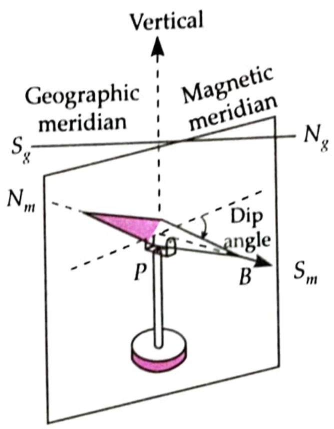

Fig. 5.30 Determination of dip at a place.

The angle of dip is different at different places on the surface of the earth. Consider a *dip needle,* which is just another compass needle but pivoted horizontally so that it is free to rotate in a vertical plane coinciding with the magnetic meridian. It orients itself so that its N-pole finally points exactly in the direction of the earth's total magnetic field $\vec{B}$. The angle between the horizontal and the final direction of the dip needle gives the angle of dip at the given location.

At the magnetic equator, the dip needle rests horizontally so that the *angle of dip is zero at the magnetic equator.* The dip needle rests vertically at the magnetic poles so that *the angle of dip is 90° at the magnetic poles.* At all other places, the dip angle lies between 0° and 90°.

3. Horizontal component of earth's magnetic field. *It is the component of the earth's total magnetic field $\vec{B}$ in the horizontal direction in the magnetic* *meridian*. If δ is the angle of dip at any place, then the horizontal component of earth's field $\vec{B}$ at that place is given by

$$B_H = B\cos\delta$$

At the magnetic equator, $\delta = 0°$, $B_H = B\cos 0° = B$

At the magnetic poles, $\delta = 90°$, $B_H = B\cos 90° = 0$

Thus the value of B_H is different at different places on the surface of the earth.

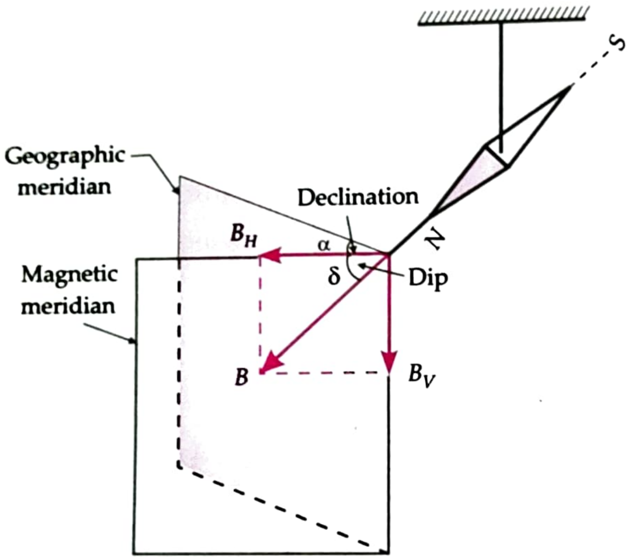

Fig. 5.31 Elements of earth's magnetic field.

Relations between elements of earth's magnetic field. Fig. 5.31 shows the three elements of earth's magnetic field. If δ is the angle of dip at any place, then the horizontal and vertical components of earth's magnetic field $\vec{B}$ at that place will be

$$B_H = B\cos\delta \quad ...(1)$$

and

$$B_V = B\sin\delta$$

$$\therefore \quad \frac{B_V}{B_H} = \frac{B\sin\delta}{B\cos\delta}$$

or

$$\frac{B_V}{B_H} = \tan\delta \quad ...(2)$$

Also

$$B_H^2 + B_V^2 = B^2(\cos^2\delta + \sin^2\delta) = B^2$$

or

$$B = \sqrt{B_H^2 + B_V^2} \quad ...(3)$$

Equations (1), (2) and (3) are the different relations between the elements of earth's magnetic field. By knowing the three elements, we can determine the magnitude and direction of the earth's magnetic field at any place.