4.8 AMPERE'S CIRCUITAL LAW AND ITS APPLICATION TO INFINITELY LONG STRAIGHT WIRE

9. *(a) State Ampere's circuital law and prove it for the magnetic field produced by a straight current-carrying conductor.*

Ampere's circuital law. Just as Gauss's law is an alternative form of Coulomb's law in electrostatics, similarly we have Ampere's circuital law as an alternative form of Biot-Savart law in magnetostatics. Ampere's circuital law gives a relationship between the line integral of a magnetic field B and the total current I which produces this field.

Ampere's circuital law states that the line integral of the magnetic field $\vec{B}$ around any closed path is equal to μ_0 (permeability constant) times the total current I threading or passing through this closed path. Mathematically,

$$\oint \vec{B} \cdot \vec{dl} = \mu_0 I$$

In a simplified form, Ampere's circuital law states that if field $\vec{B}$ is directed along the tangent to every point on the perimeter L of a closed curve and its magnitude is constant along the curve, then

$$BL = \mu_0 I$$

where I is the net current enclosed by the closed curve. The closed curve is called ***Amperean loop*** which is a *geometrical entity* and *not a* real wire loop.

Proof for a straight current-carrying conductor. Consider an infinitely long straight conductor carrying a current I. From Biot-Savart law, the magnitude of the magnetic field $\vec{B}$ due to the current-carrying conductor at a point, distant r from it is given by

$$B = \frac{\mu_0 I}{2\pi r}$$

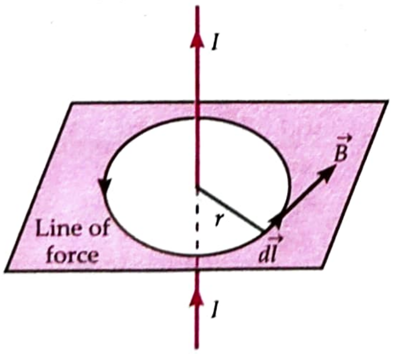

Fig. 4.48 Ampere's circuital law.

As shown in Fig. 4.48, the field $\vec{B}$ is directed along the circumference of the circle of radius r with the wire as centre. The magnitude of the field $\vec{B}$ is same for all points on the circle. To evaluate the line integral of the magnetic field $\vec{B}$ along the circle, we consider a small current element $\vec{dl}$ along the circle. At every point on the circle, both $\vec{B}$ and $\vec{dl}$ are tangential to the circle so that the angle between them is zero.

$$\therefore \quad \vec{B} \cdot \vec{dl} = B\, dl \cos 0^\circ = B\, dl$$

Hence the line integral of the magnetic field along the circular path is

$$\oint \vec{B} \cdot \vec{dl} = \oint B\, dl = B \oint dl = \frac{\mu_0 I}{2\pi r} \cdot l$$

$$= \frac{\mu_0 I}{2\pi r} \cdot 2\pi r$$

$$\therefore \quad \oint \vec{B} \cdot \vec{dl} = \mu_0 I$$

This proves Ampere's law. This law is valid for any assembly of current and for any arbitrary closed loop.

9. *(b) Calculate, using Ampere's circuital theorem, the magnetic field due to an infinitely long wire carrying a current I.*

Application of Ampere's law to a straight conductor. Fig. 4.49 shows a circular loop of radius r around an infinitely long straight wire carrying current I. As the field lines are circular, the field $\vec{B}$ at any point of the circular loop is directed along the tangent to the circle at that point. By symmetry, the magnitude of field $\vec{B}$ is same at every point of the circular loop. Therefore,

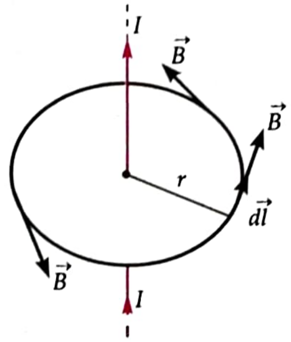

Fig. 4.49

$$\oint \vec{B} \cdot \vec{dl} = \oint B\, dl \cos 0^\circ = B \oint dl = B \cdot 2\pi r$$

From Ampere's circuital law,

$$B \cdot 2\pi r = \mu_0 I$$

$$\therefore \quad B = \frac{\mu_0 I}{2\pi r}$$

For Your Knowledge

- Ampere's circuital law is not independent of the Biot-Savart law. It can be derived from the Biot-Savart law. Its relationship to the Biot-Savart law is similar to the relationship between Gauss's law and Coulomb's law.
- Both Ampere's circuital law and Biot-Savart law relate magnetic field to the electric current.
- Ampere's and Gauss's laws relate one physical quantity (magnetic or electric quantity) on the boundary or periphery to another physical quantity (current or charge), called source, in the interior.
- Ampere's circuital law holds for steady currents which do not change with time.
- Although both Ampere's law and Biot-Savart law are equivalent in physical content, yet the Ampere's law is more useful under certain symmetrical situations. The mathematics of finding the magnetic field of a solenoid and toroid becomes much simpler if we apply Ampere's law.

4.9 MAGNETIC FIELD INSIDE A STRAIGHT SOLENOID

10. *Give a qualitative discussion of the magnetic field produced by a straight solenoid. Apply Ampere's circuital law to calculate magnetic field inside a straight solenoid.*

Magnetic field of a straight solenoid : A qualitative discussion. *A solenoid means an insulated copper wire wound closely in the form of a helix.* The word solenoid comes from a Greek word meaning *channel* and was first used by Ampere. By a long solenoid, we mean that the length of the solenoid is very large as compared to its diameter.

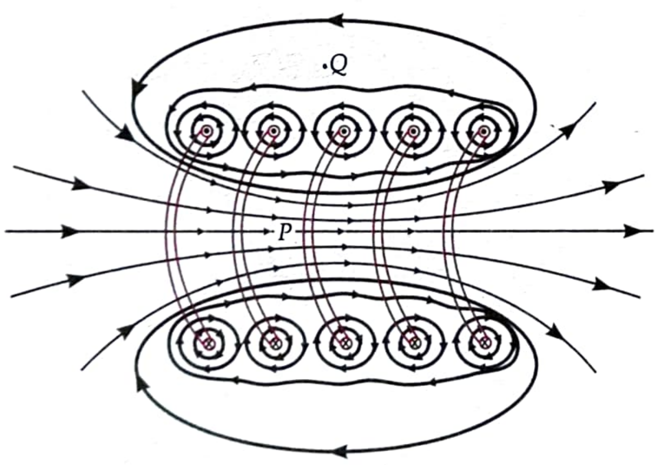

Fig 4.50 Magnetic field due to a section of a long solenoid.

Figure 4.50 shows an enlarged view of the magnetic field due to a section of a solenoid. At various turns of the solenoid, current enters the plane of paper at points marked ⊗ and leaves the plane of paper at points marked ⊙. The magnetic field at points close to a single turn of the solenoid is in the form of concentric circles like that of a straight current carrying wire. The resultant field of the solenoid is the vector sum of the fields due to all the turns of the solenoid. Obviously the fields due to the neighbouring turns add up along the axis of the solenoid but they cancel out in the perpendicular direction. At outside points such as Q, the fields of the points marked ⊗ tend to cancel out the fields of the points marked ⊙. Thus the field at interior midpoint P is uniform and strong. The field at the exterior midpoint Q is weak and is along the axis of the solenoid with no perpendicular component. Fig. 4.51 shows the field pattern of a solenoid of finite length.

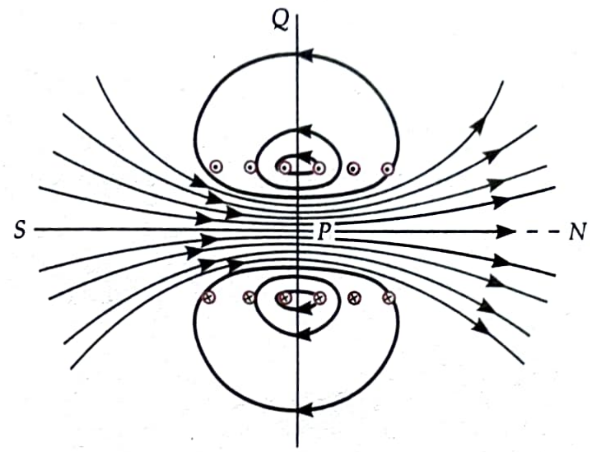

Fig 4.51 Magnetic field of a finite solenoid.

The polarity of any end of the solenoid can be determined by using *clock rule or Ampere's right hand rule.*

Ampere's right hand rule. *Grasp the solenoid with the right hand so that the fingers point along the direction of the current, the extended thumb will then indicate the face of the solenoid that has north polarity* (Fig. 4.52).

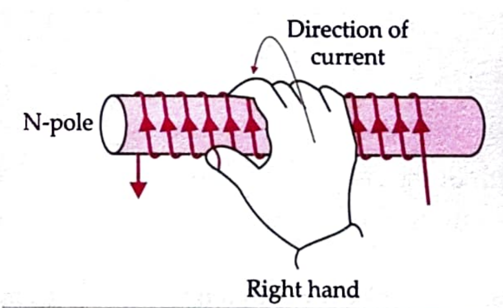

Fig. 4.52 Ampere's rule for polarity of a solenoid.

Calculation of magnetic field inside a long straight solenoid. The magnetic field inside a closely wound long solenoid is uniform everywhere and zero outside

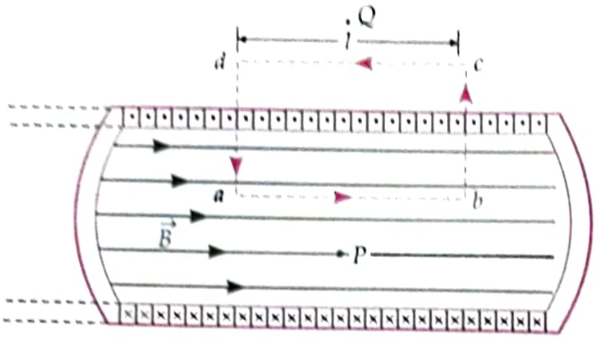

Fig. 4.53 The magnetic field of a very long solenoid.

it. Fig. 4.53 shows the sectional view of a long solenoid. At various turns of the solenoid, current comes out of the plane of paper at points marked ⊙ and enters the plane of paper at points marked ⊗. To determine the magnetic field $\vec{B}$ at any inside point, consider a rectangular closed path *abcd* as the Amperean loop. According to Ampere's circuital law,

$$\oint \vec{B}.\vec{dl}$$

$$= \mu_0 \times \text{Total current through the loop } abcd$$

Now $$\oint \vec{B}.\vec{dl} = \int_a^b \vec{B}.\vec{dl} + \int_b^c \vec{B}.\vec{dl} + \int_c^d \vec{B}.\vec{dl} + \int_d^a \vec{B}.\vec{dl}$$

But $$\int_b^c \vec{B}.\vec{dl} = \int_b^c B\,dl \cos 90° = 0$$

$$\int_d^a \vec{B}.\vec{dl} = \int_d^a B\,dl \cos 90° = 0$$

$$\int_c^d \vec{B}.\vec{dl} = 0$$

as $B = 0$ for points outside the solenoid.

$$\therefore \quad \oint \vec{B}.\vec{dl} = \int_a^b \vec{B}.\vec{dl}$$

$$= \int_a^b B\,dl \cos 0° = B\int_a^b dl = Bl$$

where,

l = length of the side *ab* of the rectangular loop *abcd*.

Let number of turns per unit length of the solenoid $= n$

Then number of turns in length l of the solenoid

$$= nl$$

Thus the current I of the solenoid threads the loop *abcd*, nl times.

$\therefore$ Total current threading the loop $abcd = nlI$

Hence $\quad Bl = \mu_0 nlI \quad$ or $\quad B = \mu_0 nI$

It can be easily shown that the magnetic field at the end of the solenoid is just one half of that at its middle. Thus

$$B_{end} = \frac{1}{2}\mu_0 nI$$

Figure 4.54 shows the variation of magnetic field on the axis of a long straight solenoid with distance x from its centre.

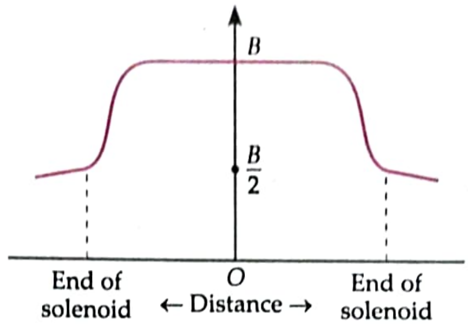

Fig. 4.54 Variation of magnetic field along the axis of solenoid.

4.10 MAGNETIC FIELD DUE TO A TOROIDAL SOLENOID

11. *Apply Ampere's circuital law to find the magnetic field both inside and outside of a toroidal solenoid.*

Magnetic field due to a toroidal solenoid. *A solenoid bent into the form of a closed ring is called a toroidal solenoid.* Alternatively, it is an anchor ring (torous) around which a large number of turns of a metallic wire are wound, as shown in Fig. 4.55. We shall see that the magnetic field $\vec{B}$ has a constant magnitude everywhere inside the toroid while it is zero in the open space interior (point P) and exterior (point Q) to the toroid.

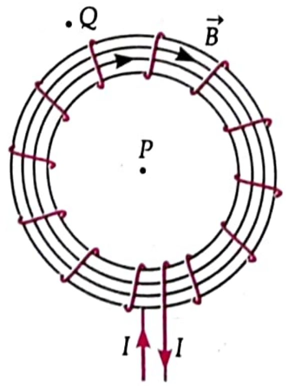

Fig. 4.55 A toroidal solenoid.

Figure 4.56 shows a sectional view of the toroidal solenoid. The direction of the magnetic field inside is clockwise as per the right-hand thumb rule for circular loops. Three circular Amperean loops are shown by

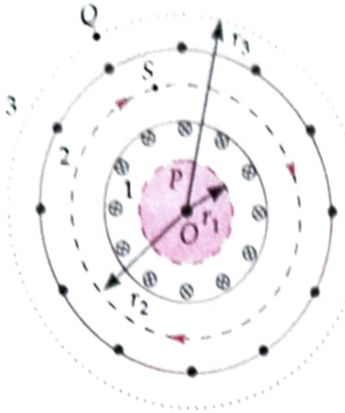

Fig. 4.56 A sectional view of the toroidal solenoid.

dashed lines. By symmetry, the magnetic field should be tangential to them and constant in magnitude for each of the loops.

1. *For points in the open space interior to the toroid.* Let B_1 be the magnitude of the magnetic field along the Amperean loop 1 of radius r_1.

Length of the loop 1, $L_1 = 2\pi r_1$

As the loop encloses no current, so $I = 0$

Applying Ampere's circuital law,

$$B_1 L_1 = \mu_0 I$$

or $$B_1 \times 2\pi r_1 = \mu_0 \times 0$$

or $$B_1 = 0$$

Thus the magnetic field at any point P in the open space interior to the toroid is zero.

2. *For points inside the toroid.* Let B be the magnitude of the magnetic field along the Amperean loop 2 of radius r.

Length of loop 2, $L_2 = 2\pi r$

If N is the total number of turns in the toroid and I the current in the toroid, then total current enclosed by the loop 2 = NI

Applying Ampere's circuital law,

$$B \times 2\pi r = \mu_0 \times NI$$

or $$B = \frac{\mu_0 NI}{2\pi r}$$

If r be the average radius of the toroid and n the number of turns per unit length, then

$$N = 2\pi r n$$

$\therefore$ $$B = \mu_0 nI$$

3. *For points in the open space exterior to the toroid.* Each turn of the toroid passes twice through the area enclosed by the Amperean loop 3. But for each turn, the current coming out of the plane of paper is cancelled by the current going into the plane of paper. Thus, $I = 0$ and hence $B_3 = 0$.