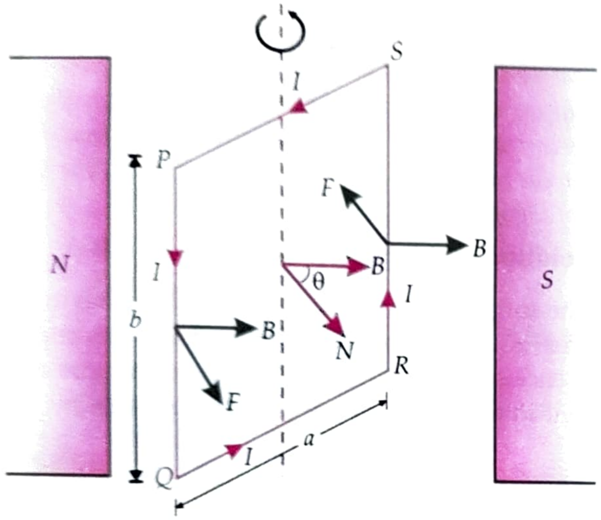
4.19 TORQUE EXPERIENCED BY A CURRENT LOOP IN A UNIFORM MAGNETIC FIELD

22. *Derive an expression for the torque acting on a current carrying loop suspended in a uniform magnetic field.*

Torque on a current loop in a uniform magnetic field. As shown in Fig. 4.92(a), consider a rectangular coil $PQRS$ suspended in a uniform magnetic field $\vec{B}$, with its axis perpendicular to the field.



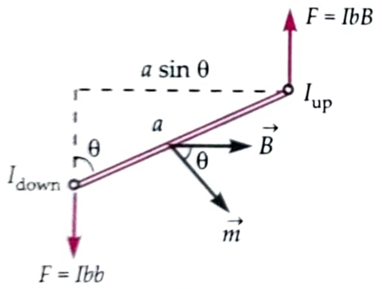


Fig. 4.92 (a) A rectangular loop $PQRS$ in a uniform magnetic field $\vec{B}$. (b) Top view of the loop, magnetic dipole moment $\vec{m}$ is shown.

Let I = current flowing through the coil $PQRS$

a, b = sides of the coil $PQRS$

$A = ab$ = area of the coil

θ = angle between the direction of $\vec{B}$ and normal to the plane of the coil.

According to Fleming's left hand rule, the magnetic forces on sides PS and QR are equal, opposite and collinear (along the axis of the loop), so their resultant is zero.

The side PQ experiences a normal inward force equal to IbB while the side RS experiences an equal normal outward force. These two forces form a couple which exerts a torque given by

$$\tau = \text{Force} \times \text{perpendicular distance}$$

$$= IbB \times a \sin\theta = IBA \sin\theta$$

If the rectangular loop has N turns, the torque increases N times *i.e.*,

$$\tau = NIBA \sin\theta$$

But $NIA = m$, the magnetic moment of the loop, so

$$\tau = mB \sin\theta$$

In vector notation, the torque $\vec{\tau}$ is given by

$$\vec{\tau} = \vec{m} \times \vec{B}$$

The direction of the torque $\vec{\tau}$ is such that it rotates the loop anticlockwise about the axis of suspension.

Special Cases

(i) When $\theta = 0°$, $\tau = 0$, *i.e.*, the torque is *minimum* when the plane of the loop is perpendicular to the magnetic field.

(ii) When $\theta = 90°$, $\tau = NIBA$, *i.e.*, the torque is *maximum* when the plane of the loop is parallel to the magnetic field. Thus

$$\tau_{max} = NIBA$$

For Your Knowledge

- The expression for torque ($\tau = NIBA \sin\theta$) holds for a planar loop of any shape. Thus the *torque* on a planar current loop depends on current, strength of magnetic field and area of the loop. It is *independent of the shape of the loop*.
- For a planar current loop of a given perimeter suspended in a magnetic field, the torque is maximum when the loop is circular in shape. This is because for a given perimeter, a circle has maximum area.
- The expression $\vec{\tau} = \vec{m} \times \vec{B}$ for the torque on a current loop in a magnetic field is analogous to the expression $\vec{\tau} = \vec{p}_e \times \vec{E}$ for the torque on an electric dipole in an electric field. This supports the fact that a *current loop is a magnetic dipole*.

- The torque on a current loop in a magnetic field is the operating principle of the electric motor and most electric meters used for measuring currents and voltages, called galvanometers.
- If the direction of the magnetic field makes an angle α with the plane of the current loop, then

$$\theta + \alpha = 90^\circ \text{ or } \theta = 90^\circ - \alpha$$

$$\therefore \quad \tau = NIBA \sin(90^\circ - \alpha) = NIBA \cos \alpha.$$

- In a uniform magnetic field, the net magnetic force on a current loop is zero but torque acting on it may be zero or non-zero.
- In a non-uniform magnetic field, the net magnetic force on a current loop is non-zero but torque acting on it may be zero or non-zero.

Examples based on Torques on Current Loops

Formulae Used

Torque on a current loop in a magnetic field,

$$\tau = NIBA \sin \theta = mB \sin \theta$$

where $m = NIA$ = magnetic dipole moment of the current loop.

In vector form, $\vec{\tau} = \vec{m} \times \vec{B}$.

Units Used

Current I is in ampere, area A in m^2, field B in tesla, torque τ in Nm and magnetic moment m in Am^2.

7. Proceed as in Exercise 4.25 on page 4.102.

4.20 MOVING COIL GALVANOMETER

23. *Describe the principle, construction and working of a pivoted-type moving coil galvanometer. Define its figure of merit.*

Moving coil galvanometer. *A galvanometer is a device to detect current in a circuit.* The commonly used moving coil galvanometer is named so because it uses a current-carrying coil that rotates (or moves) in a magnetic field due to the torque acting on it.

In a *D'Arsonval galvanometer,* the coil is suspended on a phosphor-bronze wire. It is highly sensitive and requires careful handling. In *Weston galvanometer,* the coil is pivoted between two jewellel bearings. It is rugged and portable though less sensitive, and is generally used in laboratories. The basic principle of both types of galvanometers is same.

Principle. *A current carrying coil placed in a magnetic field experiences a current dependent torque, which tends to rotate the coil and produces angular deflection.*

Construction. As shown in Fig. 4.97, a Weston (pivoted-type) galvanometer consists of a rectangular coil of fine insulated copper wire wound on a light non-magnetic metallic (aluminium) frame. The two ends of the axle of this frame are pivoted between two jewelled bearings. The motion of the coil is controlled by a pair of hair springs of phosphor-bronze. The inner ends of the springs are soldered to the two ends of the coil and the outer ends are connected to the binding screws. The springs provide the restoring torque and serve as current leads. A light aluminium pointer attached to the coil measures its deflection on a suitable scale.

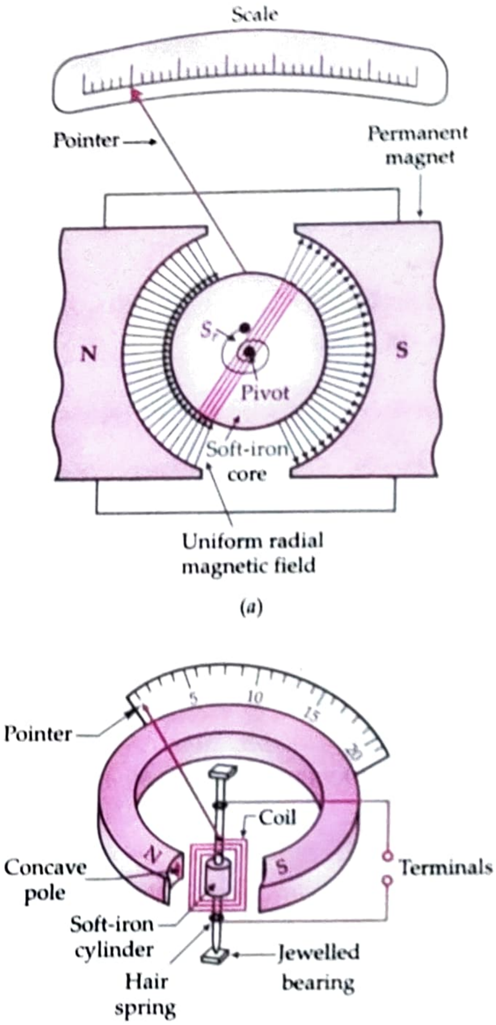


Fig. 4.97 (*a*) Top view (*b*) Front view of a pivoted-type galvanometer.

The coil is symmetrically placed between the cylindrical pole pieces of a strong permanent horse-shoe magnet.

A cylindrical soft iron core is mounted symmetrically between the concave poles of the horse-shoe magnet. This makes the lines of force pointing along the radii of a circle. Such a field is called a ***radial field***. The plane of a coil rotating in such a field remains parallel to the field in all positions, as shown in Fig. 4.97(*a*). Also, the soft iron cylinder, due to its high permeability, intensifies the magnetic field and hence increases the sensitivity of the galvanometer.

Theory and working. In Fig. 4.98(*a*), we have

I = current flowing through the coil $PQRS$

a, b = sides of the rectangular coil $PQRS$

$A = ab$ = area of the coil

N = number of turns in the coil.

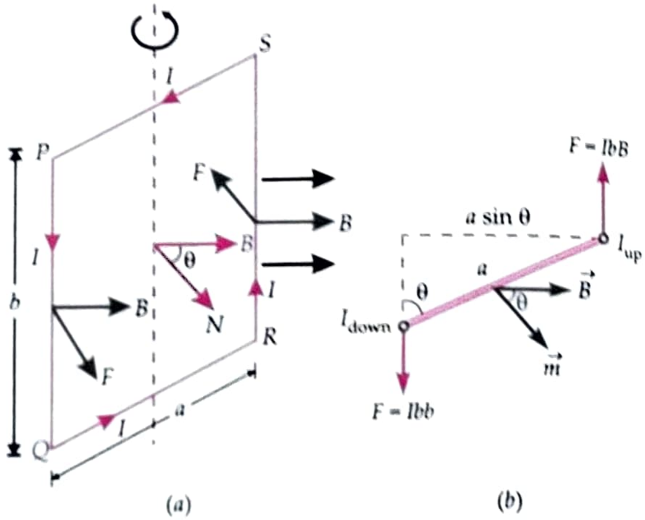


Fig. 4.98 (*a*) Rectangular loop *PQRS* in a uniform magnetic field. (*b*) Top view of the loop.

Since the field is radial, the plane of the coil always remains parallel to the field $\vec{B}$. The magnetic forces on sides PQ and SR are equal, opposite and collinear, so their resultant is zero. According to Fleming's left rule, the side PS experiences a normal inward force equal to $NIbB$ while the side QR experiences an equal normal outward force. The two forces on sides PS and QR are equal and opposite. They form a couple and exert a torque given by

$$\tau = \text{Force} \times \text{Perpendicular distance}$$

$$= NIbB \times a \sin 90° = NIB(ab) = NIBA$$

Here $\theta = 90°$, because the normal to the plane of coil remains perpendicular to the field $\vec{B}$ in all positions.

The torque τ deflects the coil through an angle α. A restoring torque is set up in the coil due to the elasticity of the springs such that

$$\tau_{restoring} \propto \alpha \quad \text{or} \quad \tau_{restoring} = k\alpha$$

where k is the ***torsion constant*** of the springs *i.e.*, torque required to produce unit angular twist. In equilibrium position,

Restoring torque = Deflecting torque

$$k\alpha = NIBA$$

or $$\alpha = \frac{NBA}{k} . I$$

or $$\alpha \propto I$$

Thus the deflection produced in the galvanometer coil is proportional to the current flowing through it. Consequently, the instrument can be provided with a scale with equal divisions along a circular scale to indicate equal steps in current. Such a scale is called ***linear scale***.

Also, $$I = \frac{k}{NBA} \cdot \alpha = G\alpha$$

The factor $G = k / NBA$ is constant for a galvanometer and is called ***galvanometer constant*** or ***current reduction factor*** of the galvanometer.

Figure of merit of a galvanometer. *It is defined as the current which produces a deflection of one scale division in the galvanometer and is given by*

$$G = \frac{I}{\alpha} = \frac{k}{NBA}$$

4.21 SENSITIVITY OF A GALVANOMETER

24. *When is a galvanometer said to be sensitive ? Define current sensitivity and voltage sensitivity of a galvanometer. State the factors on which the sensitivity of a moving coil galvanometer depends. How can we increase the sensitivity of a galvanometer ?*

Sensitivity of a galvanometer. *A galvanometer is said to be sensitive if it shows large scale deflection even when a small current is passed through it or a small voltage is applied across it.*

Current sensitivity. *It is defined as the deflection produced in the galvanometer when a unit current flows through it.*

Current sensitivity, $$I_S = \frac{\alpha}{I} = \frac{NBA}{k}$$

Voltage sensitivity. *It is defined as the deflection produced in the galvanometer when a unit potential difference is applied across its ends.*

Voltage sensitivity, $$V_S = \frac{\alpha}{V} = \frac{\alpha}{IR} = \frac{NBA}{kR}$$

Clearly, $$\text{voltage sensitivity} = \frac{\text{Current sensitivity}}{R}$$

Factors on which the sensitivity of a moving coil galvanometer depends :

1. Number of turns N in its coil.
2. Magnetic field B.
3. Area A of the coil.
4. Torsion constant k of the spring and suspension wire.

Factors by which the sensitivity of a moving coil galvanometer can be increased :

1. *By increasing the number of turns N of the coil.* But the value of N cannot be increased beyond a certain limit because that will make the galvanometer bulky and increase its resistance R.
2. *By increasing the magnetic field B.* This can be done by using a strong horse-shoe magnet and placing a soft iron core within the coil.
3. *By increasing the area A of the coil.* However, increasing A beyond a certain limit will make the galvanometer bulky and unmanageable.
4. *By decreasing the value of torsion constant k.* The torsion constant k is made small by using suspension wire and springs of phosphor bronze.

25. *Give the advantages and disadvantages of using a moving coil galvanometer.*

Advantages of a moving coil galvanometer :

1. As the deflection of the coil is proportional to the current passed through it, so a linear scale can be used to measure the deflection.
2. A moving coil galvanometer can be made highly sensitive by increasing N, B, A and decreasing k.
3. As the coil is placed in a strong magnetic field of a powerful magnet, its deflection is not affected by external magnetic fields. This enables us to use the galvanometer in any position.
4. As the coil is wound over a metallic frame, the eddy currents produced in the frame bring the coil to rest quickly.

Disadvantages of a moving coil galvanometer :

1. The main disadvantage is that its sensitiveness cannot be changed at will.
2. All types of moving coil galvanometers are easily damaged by overloading. A current greater than that which the instrument is intended to measure will burn out its hair-springs or suspension.

For Your Knowledge

- If the radial field were not present in a moving coil galvanometer, for example, if the soft iron cylinder were removed, then the torque would be $NBAI \sin \theta$ and I would be proportional $\alpha / \sin \theta$. The scale would then be *non-linear* and difficult to calibrate or to read accurately.
- Phosphor-bronze is used for suspension or hair springs because of several reasons :
  1. It is a good conductor of electricity.
  2. It does not oxidise.
  3. It is perfectly elastic.
  4. It has very little elastic after effect.
  5. It is non-magnetic.
  6. Of all materials, it has the minimum value for restoring torque per unit twist *i.e.*, smallest torsion constant k.

Following essential requirements should be met while converting a galvanometer into ammeter or voltmeter :

1. Ammeter or voltmeter should be accurate, reliable and sensitive.
2. The use of these devices in a circuit must *not alter* the current in the circuit or the potential difference across any element in the circuit.

4.23 CONVERSION OF A GALVANOMETER INTO AN AMMETER

26. *Explain how can we convert a galvanometer into an ammeter of given range.*

Conversion of a galvanometer into an ammeter. *An ammeter is a device used to measure current through a circuit element.* To measure current through a circuit element, an ammeter is connected in *series* with that element so that the current which is to be measured actually passes through it. In order to ensure that its insertion in the circuit does not change the current, an ammeter should have zero resistance. So ammeter is designed to have very small effective resistance. In fact, *an ideal ammeter should have zero resistance.*

An ordinary galvanometer is a sensitive instrument. It gives full scale deflection with a small current of few microamperes. To measure large currents with it, a small resistance is connected in parallel with the galvanometer coil. The resistance connected in this way is called a ***shunt***. Only a small part of the total current passes through the galvanometer and remaining current passes through the shunt. The value of shunt resistance depends on the range of the current required to be measured.

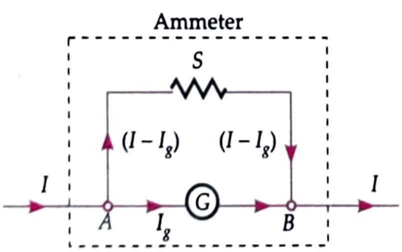


Fig. 4.99

Let G = resistance of the galvanometer

I_g = the current with which galvanometer gives full scale deflection

$0 - I$ = the required current range of the ammeter

S = shunt resistance

$I - I_g$ = current through the shunt.

As galvanometer and shunt are connected in parallel, so

P.D. across the galvanometer = P.D. across the shunt

$$I_g G = (I - I_g)\, S$$

or
$$S = \frac{I_g}{I - I_g} \times G$$

So by connecting a shunt of resistance S across the given galvanometer, we get an ammeter of desired range. Moreover,

$$I_g = \frac{S}{G+S} \times I$$

The deflection in the galvanometer is proportional to I_g and hence to I. So the scale can be graduated to read the value of current I directly.

Hence *an ammeter is a shunted or low resistance galvanometer.* Its effective resistance is

$$R_A = \frac{GS}{G+S} < S$$

27. *What is a shunt ? Mention its important uses.*

Shunt. *A shunt is a low resistance which is connected in parallel with a galvanometer (or ammeter) to protect it from strong currents.*

Uses of shunt :

1. To prevent a galvanometer from being damaged due to large current.
2. To convert a galvanometer into ammeter.
3. To increase the range of an ammeter.

For Your Knowledge

- Since an ammeter is a parallel combination of the galvanometer and the shunt resistance, so its resistance is even less than that of the shunt resistance. Moreover, $R_A << G$.
- Because of its very small resistance, an ammeter placed in a series circuit does not practically change the current in the circuit to be measured.
- The resistance of an ideal ammeter is zero.
- Higher the range of ammeter to be prepared from a given galvanometer, lower is the value of the shunt resistance required for the purpose.
- The ammeter of lower range has a higher resistance than the ammeter of higher range.
- The range of an ammeter can be increased but it cannot be decreased.

4.24 CONVERSION OF A GALVANOMETER INTO A VOLTMETER

28. *Explain how can we convert a galvanometer into a voltmeter of given range.*

Conversion of a galvanometer into a voltmeter. *A voltmeter is a device for measuring potential difference across any two points in a circuit.* It is connected in parallel with the circuit element across which the potential difference is intended to be measured. As a result, a small part of the total current passes through the voltmeter and so the current through the circuit element decreases. This decreases the potential difference required to be measured. To avoid this, the voltmeter should be designed to have very high resistance. In fact, *an ideal voltmeter should have infinite resistance.*

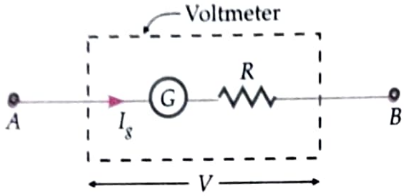


Fig. 4.100

A galvanometer can be converted into a voltmeter by connecting a high resistance in series with it. The value of this resistance is so adjusted that only current I_g which produces full scale deflection in the galvanometer, passes through the galvanometer.

Let

G = resistance of the galvanometer

I_g = the current with which galvanometer gives full scale deflection

$0 - V$ = required range of the voltmeter, and

R = the high series resistance which restricts the current to safe limit I_g.

$\therefore$ Total resistance in the circuit = $R + G$

By Ohm's law,

$$I_g = \frac{\text{Potential difference}}{\text{Total resistance}} = \frac{V}{R+G}$$

or $$R + G = \frac{V}{I_g} \quad \text{or} \quad R = \frac{V}{I_g} - G$$

So by connecting a high resistance R in series with the galvanometer, we get a voltmeter of desired range. Moreover, the deflection in the galvanometer is proportional to current I_g and hence to V. The scale can be graduated to read the value of potential difference directly.

Hence *a voltmeter is a high resistance galvanometer.* Its effective resistance is

$$R_V = R + G >> G$$