4.6 MAGNETIC FIELD AT THE CENTRE OF CIRCULAR CURRENT LOOP

7. Apply Biot-Savart law to derive an expression for the magnetic field at the centre of a current-carrying circular loop.

Magnetic field at the centre of a circular current loop. As shown in Fig. 4.23, consider a circular loop of wire of radius r carrying current I. We wish to calculate its magnetic field at the centre O. The entire loop can be divided into a large number of small current elements.

Consider a current element $\vec{dl}$ of the loop. According to Biot-Savart law, the magnetic field at the centre O due to this element is

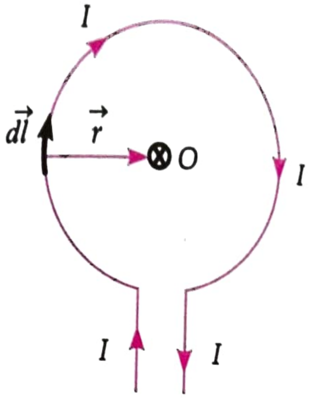


Fig. 4.23 Magnetic field at the centre of a circular current loop.

$$\vec{dB} = \frac{\mu_0 I}{4\pi} \cdot \frac{\vec{dl} \times \vec{r}}{r^3}$$

The field at point O points normally into the plane of paper, as shown by encircled cross ⊗. The direction of $\vec{dl}$ is along the tangent, so $\vec{dl} \perp \vec{r}$. Consequently, the magnetic field at the centre O due to this current element is

$$dB = \frac{\mu_0 I}{4\pi} \frac{dl \sin 90°}{r^2} = \frac{\mu_0 I}{4\pi} \cdot \frac{dl}{r^2}$$

The magnetic field due to all such current elements will point into the plane of paper at centre O. Hence the total magnetic field at the centre O is

$$B = \int dB = \int \frac{\mu_0 I}{4\pi} \cdot \frac{dl}{r^2} = \frac{\mu_0 I}{4\pi r^2} \int dl$$

$$= \frac{\mu_0 I}{4\pi r^2} \cdot l = \frac{\mu_0 I}{4\pi r^2} \cdot 2\pi r$$

or
$$B = \frac{\mu_0 I}{2r}$$

If instead of a single loop, there is a coil of N turns, all wound over one another, then

$$B = \frac{\mu_0 N I}{2r}.$$

4.7 MAGNETIC FIELD ON THE AXIS OF A CIRCULAR CURRENT LOOP

8. *Apply Biot-Savart law to find the magnetic field due to a circular current-carrying loop at a point on the axis of the loop. State the rules used to find the direction of this magnetic field.*

Magnetic field along the axis of a circular current loop. Consider a circular loop of wire of radius a and carrying current I, as shown in Fig. 4.24. Let the plane of the loop be perpendicular to the plane of paper. We wish to find field $\vec{B}$ at an axial point P at a distance r from the centre C.

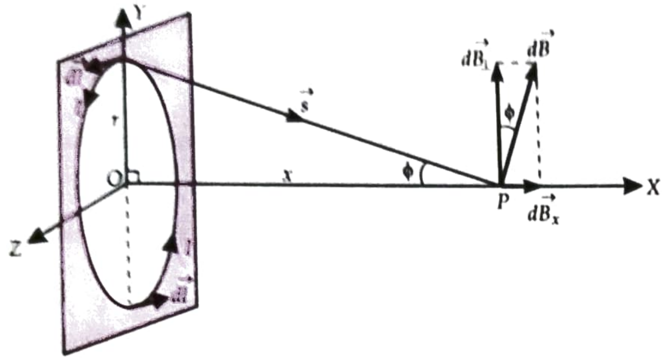


Fig. 4.24 Magnetic field on the axis of a circular current loop.

Consider a current element $d\vec{l}$ at the top of the loop. It has an outward coming current.

If $\vec{s}$ be the position vector of point P relative to the element $d\vec{l}$, then from Biot-Savart law, the field at point P due to the current element is

$$dB = \frac{\mu_0}{4\pi} \cdot \frac{I\, dl \sin\theta}{s^2}$$

Since $d\vec{l} \perp \vec{s}$, *i.e.*, $\theta = 90°$, therefore

$$dB = \frac{\mu_0}{4\pi} \cdot \frac{I\, dl}{s^2}$$

The field $d\vec{B}$ lies in the plane of paper and is perpendicular to $\vec{s}$, as shown by $\vec{PQ}$. Let ϕ be the angle between OP and CP. Then dB can be resolved into *two* rectangular components.

1. $dB \sin\phi$ along the axis,
2. $dB \cos\phi$ perpendicular to the axis.

For any two diametrically opposite elements of the loop, the components perpendicular to the axis of the loop will be equal and opposite and will cancel out. Their axial components will be in the same direction, *i.e.*, along CP and get added up.

∴ Total magnetic field at the point P in the direction CP is

$$B = \int dB \sin\phi$$

But $\sin\phi = \frac{a}{s}$ and $dB = \frac{\mu_0}{4\pi} \cdot \frac{I\, dl}{s^2}$

$$\therefore \quad B = \int \frac{\mu_0}{4\pi} \cdot \frac{I\, dl}{s^2} \cdot \frac{a}{s}$$

Since μ_0 and I are constant, and s and a are same for all points on the circular loop, we have

$$B = \frac{\mu_0 Ia}{4\pi s^3} \int dl = \frac{\mu_0 Ia}{4\pi s^3} \cdot 2\pi a = \frac{\mu_0 Ia^2}{2s^3}$$

$$\left[\because \int dl = \text{circumference} = 2\pi a\right]$$

or $$B = \frac{\mu_0 Ia^2}{2(r^2 + a^2)^{3/2}} \qquad \left[\because s = (r^2 + a^2)^{1/2}\right]$$

As the direction of the field is along +ve X-direction, so we can write

$$\vec{B} = \frac{\mu_0 Ia^2}{2(r^2 + a^2)^{3/2}} \hat{i}$$

If the coil consists of N turns, then

$$B = \frac{\mu_0 N Ia^2}{2(r^2 + a^2)^{3/2}}$$

Special Cases

1. **At the centre of the current loop,** $r = 0$, therefore

$$B = \frac{\mu_0 N Ia^2}{2a^3} = \frac{\mu_0 NI}{2a}$$

or $$B = \frac{\mu_0 N IA}{2\pi a^3}$$

where $A = \pi a^2$ = area of the circular current loop. The field is directed perpendicular to the plane of the current loop.

2. **At the axial points lying far away from the coil,** $r >> a$, so that

$$B = \frac{\mu_0 N I a^2}{2r^3} = \frac{\mu_0 N IA}{2\pi r^3}$$

This field is directed along the axis of the loop and falls off as the cube of the distance from the current loop.

3. **At an axial point at a distance equal to the radius of the coil** *i.e.*, $r = a$, we have

$$B = \frac{\mu_0 N I a^2}{2(a^2 + a^2)^{3/2}} = \frac{\mu_0 NI}{2^{5/2} a}.$$

Direction of the magnetic field. Fig. 4.25 shows the magnetic lines of force of a circular wire carrying current. The lines of force near the wire are almost concentric circles. As we move radially towards the

* Ampere's law →

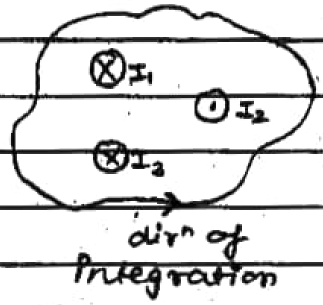
The line integral of magnetic field in a closed loop is always equal to the product of permeability of free space and summation of current in the closed loop.

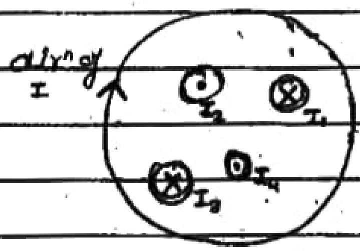
$$\oint \vec{B} \cdot d\vec{l} = \mu_0 \Sigma I$$

$\mu_0 \rightarrow$ permeability of free space

$\mu_0 = 4\pi \times 10^{-7} \frac{\text{Henry}}{\text{metre}}$

NOTE → If we curl our fingers of right hand in the dir^n of integration then thumb will represent dir^n of positive current.

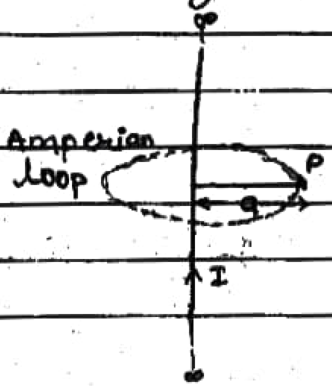


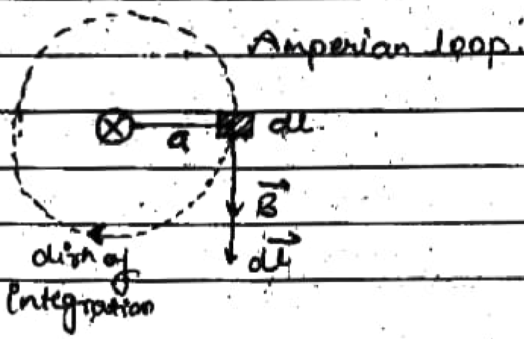


$$\oint \vec{B} \cdot d\vec{l} = \mu_0 [I_2 - I_1 - I_3]$$

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 [I_1 + I_3 - I_2 - I_4]$$

Application # 01 → To determine magnetic field intensity of infinite long wire using Ampere's law.





$$\oint \vec{B} \cdot \vec{dl} = \mu_0 \Sigma I$$

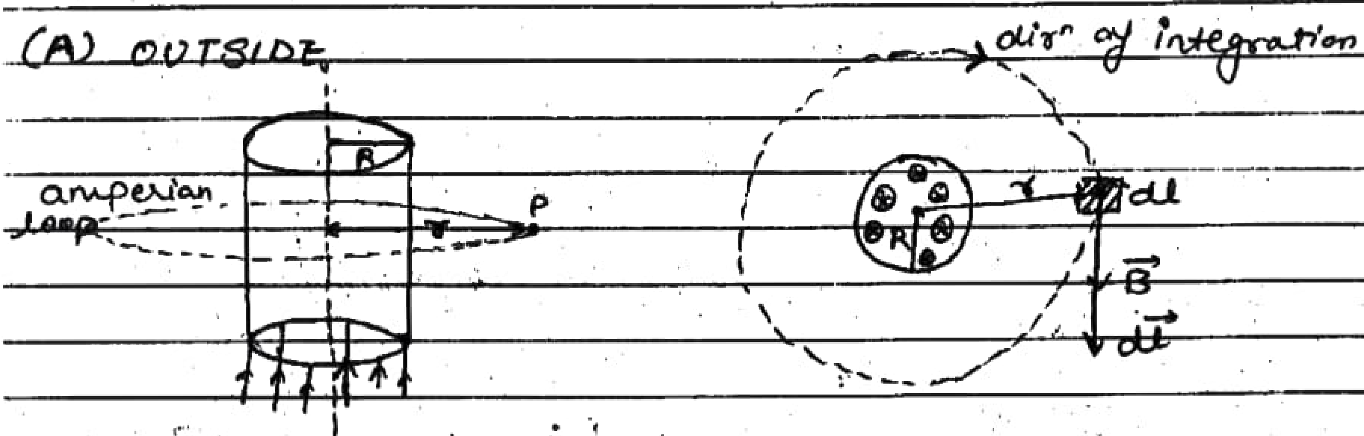
$$\oint B \, dl \cos 0 = \mu_0 [+I]$$

$$B (2\pi a) = \mu_0 I$$

$$B = \frac{\mu_0 I}{2\pi a}$$

Application # 02 → Magnetic field intensity due to long current carrying solid cylinder.

(A) OUTSIDE



$$\oint \vec{B} \cdot \vec{dl} = \mu_0 \Sigma I$$

$$\oint B \, dl \cos 0 = \mu_0 [I]$$

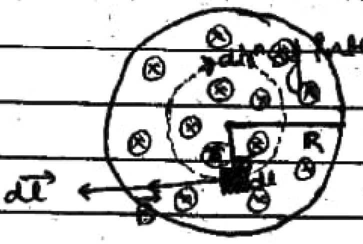
$$B (2\pi r) = \mu_0 I$$

$$\boxed{B = \frac{\mu_0 I}{2\pi r}}$$

$$\boxed{B = \frac{\mu_0 I}{2\pi R}}$$ (on the surface)

(B) INSIDE

$$\oint \vec{B} \cdot \vec{dl} = \mu_0 \Sigma I$$



$\pi R^2 \longrightarrow I$

$1 \longrightarrow \dfrac{I}{\pi R^2}$

$\pi r^2 \longrightarrow I \dfrac{\pi r^2}{\pi R^2} = \dfrac{I r^2}{R^2}$

$$\oint B\,dl \cos 0^\circ = \mu_0 \left[\frac{r^2}{R^2} I\right]$$

$$B(2\pi r) = \frac{\mu_0 I r^2}{R^2}$$

$$\boxed{B = \frac{\mu_0 I r}{2\pi R^2}}$$

* Graph of magnetic field v/s radius of the loop (distance of point P from center) →