EXERCISE 12.2

Unless stated otherwise, take $\pi = \frac{22}{7}$.

1 Find the area of a sector of a circle with radius 6 cm, if angle of the sector is 60°.

Sol. We know that, area of sector of a circle $= \frac{\theta}{360^\circ} \times \pi r^2$

Given, radius of circle, $r = 6$ cm
and angle of sector, $\theta = 60^\circ$

$\therefore$ Area of sector of a circle $= \frac{60^\circ}{360^\circ} \times \frac{22}{7} \times (6)^2 = \frac{132}{7}$ cm^2

2 Find the area of a quadrant of a circle whose circumference is 22 cm.

Sol. Given, circumference of a circle = 22 cm

$\Rightarrow \quad 2\pi r = 22 \Rightarrow 2 \times \frac{22}{7} \times r = 22$

$\Rightarrow \quad r = \frac{22 \times 7}{2 \times 22} = \frac{7}{2}$ cm

Now, area of a quadrant of a circle $= \frac{\pi r^2}{4}$

$$= \frac{\frac{22}{7} \times \left(\frac{7}{2}\right)^2}{4}$$

$$= \frac{11}{14} \times \frac{49}{4} = \frac{77}{8} \text{ cm}^2$$

3 The length of the minute hand of a clock is 14 cm. Find the area swept by the minute hand in 5 min.

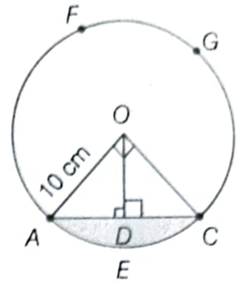
Sol. Similar to same as Example 1 of Topic 2. [**Ans.** $\frac{154}{3}$ cm^2]

4 A chord of a circle of radius 10 cm subtends a right angle at the centre. Find the area of the corresponding

(*i*) minor segment.

(*ii*) major sector. [Take, $\pi = 3.14$]

Sol. Given, radius of a circle, $AO = 10$ cm and $\angle AOC = 90^\circ$



Area of $\Delta AOC = \frac{1}{2} \times OA \times OC$

$= \frac{1}{2} \times 10 \times 10 = 50$ cm^2

Area of sector $OAECO = \frac{\theta}{360^\circ} \times \pi r^2$

$= \frac{90^\circ}{360^\circ} \times 3.14 \times (10)^2$

$= \frac{314}{4} = 78.5$ cm^2

(*i*) Area of minor segment $AECDA$
$=$ Area of sector $OAECO$ $-$ Area of $\Delta\ AOC$
$= 78.5 - 50 = 28.5$ cm^2

(*ii*) Area of major sector $OAFGCO$
$=$ Area of circle $-$ Area of sector $OAECO$
$= 3.14 \times (10)^2 - 78.5$
$= 314 - 78.5 = 235.5$ cm^2

5 In a circle of radius 21 cm, an arc subtends an angle of 60° at the centre. Find

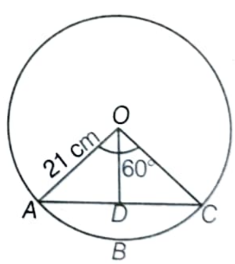
(*i*) the length of the arc.

(*ii*) area of the sector formed by the arc.

(*iii*) area of the segment formed by the corresponding chord.

Sol. Given, arc ABC subtends an angle 60° at the centre.
$\therefore \quad \theta = 60^\circ$ and radius, $r = 21$ cm

(*i*) Length of arc ABC



$= \frac{\theta}{360^\circ} \times 2\pi r$

$= \frac{60^\circ}{360^\circ} \times 2 \times \frac{22}{7} \times 21$

$= \frac{44}{6} \times 3 = 22$ cm

(*ii*) Area of the sector formed by the arc

$= \frac{60^\circ}{360^\circ} \times \frac{22}{7} \times (21)^2$

$= \frac{22}{6 \times 7} \times 21 \times 21 = \frac{9702}{42} = 231$ cm^2

(*iii*) Given, $\angle AOC = 60°$ and $OA = OC$ (radii of circle)

Let $\angle OAC = \angle OCA = x$

In ΔOAC, $\angle OAC + \angle AOC + \angle OCA = 180°$

$\Rightarrow \quad 60° + x + x = 180° \Rightarrow 60° + 2x = 180$

$\Rightarrow \quad 2x = 120 \Rightarrow x = 60°$

$\therefore \quad \angle A = \angle O = \angle C = 60°$

$\Rightarrow \Delta OAC$ is an equilateral triangle.

So, area of $\Delta OAC = \frac{\sqrt{3}}{4} \times (21)^2 = \frac{441\sqrt{3}}{4}$ cm^2

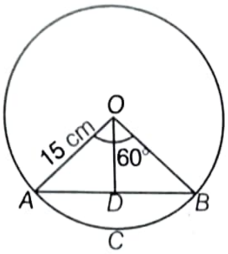
$$\left[\because \text{area of equilateral triangle} = \frac{\sqrt{3}}{4}(\text{side})^2\right]$$

Hence, area of the segment

= Area of sector formed by the arc − Area of ΔOAC

$= \left(231 - \frac{441\sqrt{3}}{4}\right)$ cm^2

6 A chord of a circle of radius 15 cm subtends an angle of 60° at the centre. Find the areas of the corresponding minor and major segments of the circle. [Take, $\pi = 3.14$ and $\sqrt{3} = 1.73$]



Sol. Given, chord AB subtends an angle 60° at the centre.

$\therefore \quad \angle AOB = 60°$ and $OA = OB$ [radii of circle]

$\therefore \quad \angle OBA = \angle OAB = x$ [say]

[$\because$ angles opposite to equal sides are also equal]

In ΔAOB,

$\angle AOB + \angle OBA + \angle OAB = 180°$

[by angle sum property of triangle]

$\Rightarrow \quad 60° + x + x = 180° \Rightarrow 2x = 180° - 60°$

$\Rightarrow \quad x = \frac{120°}{2} = 60°$

$\Rightarrow \quad \angle OBA = \angle OAB = 60° = \angle AOB$

$\therefore \Delta AOB$ is an equilateral triangle.

Now, area of $\Delta AOB = \frac{\sqrt{3}}{4} \times (15)^2$

$$\left[\because \text{area of equilateral triangle} = \frac{\sqrt{3}}{4}(\text{side})^2\right]$$

$= 56.25 \times 1.73 = 97.3125$ cm^2

Area of sector $OACBO = \frac{\theta}{360°} \times \pi r^2 = \frac{60°}{360°} \times 3.14 \times (15)^2$

$= \frac{3.14 \times 225}{6} = \frac{706.5}{6} = 117.75$ cm^2

$\therefore$ Area of minor segment $ACBDA$

= Area of sector $OACBO$ − Area of ΔAOB

$= 117.75 - 97.3125 = 20.4375$ cm^2

Now, area of major segment

= Area of circle − Area of minor segment

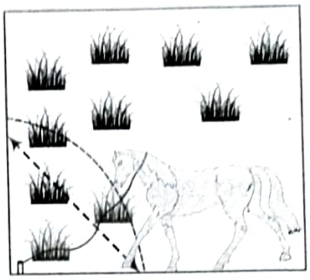
$= \pi(15)^2 - 20.4375 = 3.14 \times 225 - 20.4375$

$= 706.5 - 20.4375 = 686.0625$ cm^2

7 A chord of a circle of radius 12 cm subtends an angle of 120° at the centre. Find the area of the corresponding segment of the circle. [Take $\pi = 3.14$ and $\sqrt{3} = 1.73$]

Sol. Similar to Example 3 of Topic 2 [Ans. 88.44 cm^2]

8 A horse is tied to a peg at one corner of a square shaped grass field of side 15 m by means of a 5 m long rope (see the figure). Find



(*i*) the area of the part of the field in which the horse can graze.

(*ii*) the increase in the grazing area if the rope were 10 m long instead of 5 m. [Take, $\pi = 3.14$]

Sol. Given, side of a square = 15 m

$\therefore$ Area of square $= (15)^2 = 225$ m^2

[$\because$ area of square = (side)^2]

Also given, length of rope = 5 m

$\therefore \quad$ Radius of arc = 5 m

(*i*) Area of the field graze by the horse,

$A_1 = \frac{\theta}{360°} \times \pi r^2 = \frac{90°}{360°} \times 3.14 \times (5)^2$

[$\because$ each angle of a square is 90°]

$= \frac{3.14 \times 25}{4} = \frac{78.5}{4} = 19.625$ cm^2

(*ii*) If length of rope = 10 m = r_1 (say)

Then, area of the field graze by the horse,

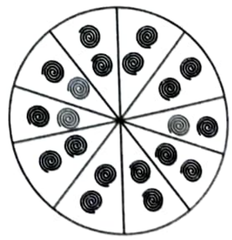
$A_2 = \frac{\theta}{360°} \times \pi r_1^2 = \frac{90°}{360°} \times 3.14 \times (10)^2$

$= \frac{3.14 \times 100}{4} = \frac{314}{4} = 78.5$ cm^2

$\therefore$ Required increase in the grazing area

$= A_2 - A_1 = 78.5 - 19.625 = 58.875$ cm^2

9 A brooch is made with silver wire in the form of a circle with diameter 35 mm. The wire is also used in making 5 diameters which divide the circle into 10 equal sectors as shown in the figure. Find



(*i*) the total length of the silver wire required.

(*ii*) the area of each sector of the brooch.

Sol. Given, diameter of circle, $d = 35$ mm

$\therefore$ Circumference of circle $= \pi d$ $\quad [\because d = 2r]$

$$= \frac{22}{7} \times 35 = 110 \text{ mm}^2$$

Now, length of 5 diameters $= 5 \times 35 = 175$ mm

(i) Total length of the silver wire $= \pi d + 5d$

$$= 110 + 175 = 285 \text{ mm}^2$$

(ii) Here, we see that total circle is divided into 10 sectors.

$\therefore$ Angle of each sector $= \frac{360^\circ}{10} = 36^\circ$

Then, area of each sector of the brooch $= \frac{\theta}{360^\circ} \times \pi r^2$

$$= \frac{36^\circ}{360^\circ} \times \frac{22}{7}\left(\frac{35}{2}\right)^2 \quad \left[\because r = \frac{d}{2} = \frac{35}{2} \text{ mm}\right]$$

$$= \frac{1}{10} \times \frac{22}{1} \times \frac{5}{2} \times \frac{35}{2} = \frac{11 \times 35}{2 \times 2} = \frac{385}{4} \text{ mm}^2$$

10 An umbrella has 8 ribs which are equally spaced (see the figure). Assuming umbrella to be a flat circle of radius 45 cm, find the area between the two consecutive ribs of the umbrella.

Sol. Given, umbrella to be a flat circle. So, the central angle of an umbrella is 360°.

Since, umbrella has 8 ribs.

$\therefore$ Angle between two ribs $= \frac{360^\circ}{8} = 45^\circ$

Area between two ribs

= Area of one sector of the umbrella

$$= \frac{\theta}{360^\circ} \times \pi r^2 = \frac{45^\circ}{360^\circ} \times \frac{22}{7} \times (45)^2 \quad [\because r = 45, \text{ given}]$$

$$= \frac{22}{7 \times 8}(45)^2 = \frac{22275}{28} \text{ cm}^2$$

11 A car has two wipers which do not overlap. Each wiper has a blade of length 25 cm sweeping through an angle of 115°. Find the total area cleaned at each sweep of the blades.

Sol. Given, length of wiper blade $= 25$ cm $= r$ (say)

and angle made by this blade, $\theta = 115^\circ$

$\therefore$ Area cleaned by one blade

= Area of sector formed by blade

$$= \frac{\theta}{360^\circ} \times \pi r^2 = \frac{115^\circ}{360^\circ} \times \frac{22}{7} \times (25)^2$$

$$= \frac{23 \times 22}{72 \times 7} \times 625 = \frac{23 \times 11 \times 625}{36 \times 7} = \frac{158125}{252} \text{ cm}^2$$

$\therefore$ Total area cleaned by both blades

= 2 × Area cleaned by one blade

$$= \frac{2 \times 158125}{252} = \frac{158125}{126} \text{ cm}^2$$

12 To warn ships for underwater rocks, a light house spreads a red coloured light over a sector of angle 80° to a distance of 16.5 km. Find the area of the sea over which the ships are warned. [Take, $\pi = 3.14$]

Sol. Given, sector angle, $\theta = 80^\circ$

and distance or radius, $r = 16.5$ km

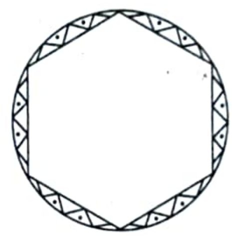
$$\therefore \text{ Area of sector} = \frac{\theta}{360^\circ} \times \pi r^2 = \frac{80^\circ}{360^\circ} \times 3.14 \times (16.5)^2$$

$$= \frac{2 \times 3.14 \times 272.25}{9}$$

$$= \frac{1709.73}{9} = 189.97 \text{ km}^2$$

which is the required area of the sea over which the ships are warned.

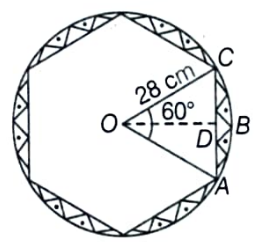
13 A round table cover has six equal designs as shown in the figure. If the radius of the cover is 28 cm, find the cost of making the designs at the rate of ₹ 0.35 per cm². [take, $\sqrt{3} = 1.732$]



Sol. We know that the central angle of a circle is 360°.

$\therefore$ Angle of each sector

$$= \frac{360^\circ}{6} = 60^\circ$$



$\therefore$ $\angle AOC = 60^\circ$

and $OA = OC$ $\quad$ [radii of circle]

$\therefore$ $\angle OAC = \angle OCA = x$ (say)

[$\because$ angles opposite to equal sides of a triangle are also equal]

In ΔAOC, $\angle AOC + \angle OAC + \angle OCA = 180^\circ$

[by angle sum property of triangle]

$\Rightarrow$ $60^\circ + x + x = 180^\circ$

$\Rightarrow$ $2x = 180^\circ - 60^\circ$

$\Rightarrow$ $x = \frac{120^\circ}{2}$

$\Rightarrow$ $x = 60^\circ$

$\Rightarrow$ $\angle OAC = \angle OCA = 60^\circ = \angle AOC$

$\therefore$ ΔAOC is an equilateral triangle.

Area of $\Delta AOC = \frac{\sqrt{3}}{4} \times (28)^2 = 333.2 \text{ cm}^2$

[$\because$ area of an equilateral triangle $= \frac{\sqrt{3}}{4}(\text{side})^2$]

Now, area of sector $OABCO = \frac{\theta}{360^\circ} \times \pi r^2$

$$= \frac{60^\circ}{360^\circ} \times \frac{22 \times (28)^2}{7} = \frac{22 \times 4 \times 28}{6} = 410.67 \text{ cm}^2$$

$\therefore$ Area of segment $ABCA$

= Area of sector $OABCO$ − Area of $\Delta\, AOC$

$= 410.67 - 333.2 = 77.47 \text{ cm}^2$

Now, area of six segments $= 6 \times 77.47 = 464.82 \text{ cm}^2$

Since, the cost of making the design is ₹ 0.35 per cm^2.

$\therefore$ Total cost $= 464.82 \times 0.35 =$ ₹ 162.69

14 Tick the correct answer in the following question. Area of a sector of angle p (in degrees) of a circle with radius R is

(i) $\frac{p}{180^\circ} \times 2\pi R$ (ii) $\frac{p}{180^\circ} \times \pi R^2$

(iii) $\frac{p}{360^\circ} \times 2\pi R$ (iv) $\frac{p}{720^\circ} \times 2\pi R^2$

Sol. (iv) Given, sector angle $= p$ and radius of circle $= R$

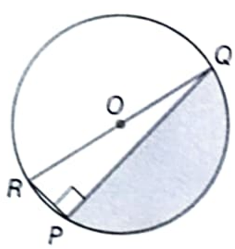
$\therefore$ Area of sector $= \frac{\theta}{360^\circ} \times \pi r^2 = \frac{p}{360^\circ} \times \pi R^2$

$$= \frac{p}{720^\circ} \times 2\pi R^2$$

EXERCISE 12.3

Unless stated otherwise, take $\pi = \frac{22}{7}$.

1 Find the area of the shaded region in the given figure, if $PQ = 24$ cm, $PR = 7$ cm and O is the centre of the circle.



Sol. Given, $PQ = 24$ cm and $PR = 7$ cm

We know that any angle made by the diameter RQ in the semi-circle is 90°.

$\therefore \quad \angle RPQ = 90^\circ$

In right angled ΔRPQ,

$RQ^2 = PR^2 + PQ^2$ [by Pythagoras theorem]

$\Rightarrow \quad RQ^2 = 7^2 + 24^2 \Rightarrow RQ^2 = 49 + 576$

$\Rightarrow \quad RQ^2 = 625$

$\Rightarrow \quad RQ = \sqrt{625}$ [$\because$ side cannot be negative]

$\Rightarrow \quad RQ = 25$ cm

$\therefore$ Area of right angled $\Delta RPQ = \frac{1}{2} \times RP \times PQ$

[$\because$ area of triangle $= \frac{1}{2} \times$ base $\times$ height]

$$= \frac{1}{2} \times 7 \times 24 = 84 \text{ cm}^2$$

Area of semi-circle

$= \frac{\pi r^2}{2} = \frac{22}{7 \times 2}\left(\frac{25}{2}\right)^2$ $\left[\because r = \frac{RQ}{2} = \frac{25}{2} \text{ cm}\right]$

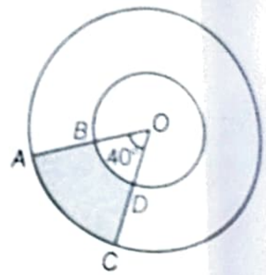
$= \frac{11 \times 625}{28} = \frac{6875}{28} \text{ cm}^2$

Hence, area of the shaded region

= Area of the semi-circle − Area of right angled ΔRPQ

$= \frac{6875}{28} - 84 = \frac{6875 - 2352}{28} = \frac{4523}{28} \text{ cm}^2$

2 Find the area of the shaded region in figure, if radii of the two concentric circles with centre O are 7 cm and 14 cm respectively and $\angle AOC = 40^\circ$.



Sol. Given, $OB = 7 \text{ cm} = r$ (say)

$OA = 14 \text{ cm} = r_1$ (say), and $\angle AOC = 40^\circ$

$\therefore \angle BOD = \angle AOC = 40^\circ = \theta$ (say) [from figure]

Now, area of sector $BOD = \frac{\theta}{360^\circ} \times \pi r^2$

$= \frac{40^\circ}{360^\circ} \times \frac{22}{7} \times (7)^2 = \frac{154}{9} \text{ cm}^2$

and area of sector $AOC = \frac{\theta}{360^\circ} \times \pi r_1^2$

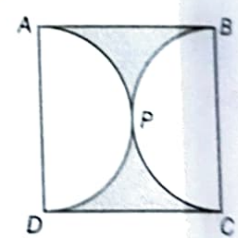
$= \frac{40^\circ}{360^\circ} \times \frac{22}{7} \times (14)^2 = \frac{22 \times 28}{9} = \frac{616}{9} \text{ cm}^2$

Hence, required area of shaded region

= Area of sector AOC − Area of sector BOD

$= \frac{616}{9} - \frac{154}{9} = \frac{462}{9} = \frac{154}{3} \text{ cm}^2$

3 Find the area of the shaded region in figure, if $ABCD$ is a square of side 14 cm and APD and BPC are semi-circles.



Sol. Given, side of square = 14 cm

Also, APD and BPC are semi-circles, therefore

Their radius, $r = \frac{14}{2} = 7$ cm.

Now, area of semi-circle APD

= Area of semi-circle $BPC = \frac{\pi r^2}{2}$

$= \frac{22}{7 \times 2}(7)^2 = 77 \text{ cm}^2$

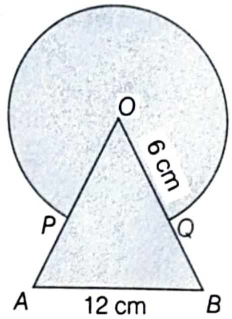
and area of square $ABCD = (\text{Side})^2 = (14)^2 = 196 \text{ cm}^2$

Hence, area of shaded region

= Area of square − (Area of semi-circle APD + Area of semi-circle BPC)

$= 196 - (77 + 77) = 42 \text{ cm}^2$

4 Find the area of the shaded region in figure, where a circular arc of radius 6 cm has been drawn with vertex O of an equilateral triangle OAB of side 12 cm as centre.



Sol. Given, radius of circle, $r = 6$ cm

and side of equilateral triangle $= 12$ cm

Since, OAB is an equilateral triangle.

$\therefore \quad \angle O = \angle A = \angle B = 60°$

Area of an equilateral $\Delta OAB = \dfrac{\sqrt{3}}{4}(\text{Side})^2$

$$= \frac{\sqrt{3}}{4}(12)^2 = 36\sqrt{3}\ \text{cm}^2$$

Area of circle outside the triangle

$= $ Area of circle $-$ Area of sector $OPQO$

$$= \pi r^2 - \frac{60°}{360°} \times \pi r^2 = \pi r^2 - \frac{\pi r^2}{6} = \frac{5}{6}\pi r^2$$

$$= \frac{5}{6}\left[\frac{22}{7} \times (6)^2\right] = \frac{110 \times 36}{6 \times 7} = \frac{660}{7}\ \text{cm}^2$$

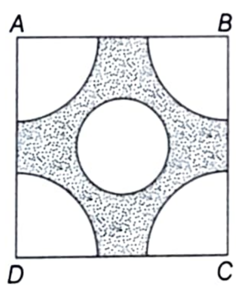
Hence, total area of the shaded region

$= $ Area of an equilateral ΔOAB

$+$ Area of circle outside the triangle

$$= \left(36\sqrt{3} + \frac{660}{7}\right)\text{cm}^2$$

5 From each corner of a square of side 4 cm, a quadrant of a circle of radius 1 cm is cut and also a circle of diameter 2 cm is cut as shown in figure. Find the area of the remaining portion of the square.



Sol. Given, side of a square $= 4$ cm

Radius of quadrant, $r_1 = 1$ cm

and radius of circle of diameter 2 cm, $r_2 = \dfrac{2}{2} = 1$ cm

Area of one quadrant $= \dfrac{\pi r_1^2}{4} = \dfrac{22}{7 \times 4}(1)^2 = \dfrac{22}{28}$ cm^2

Then, area of four quadrants $= 4\left(\dfrac{22}{28}\right) = \dfrac{22}{7}$ cm^2

Now, area of circle $= \pi r_2^2 = \dfrac{22}{7}(1)^2 = \dfrac{22}{7}$ cm^2

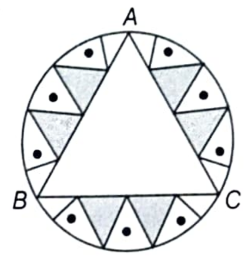
and area of square $ABCD = (\text{Side})^2 = (4)^2 = 16$ cm^2

$\therefore$ Area of shaded region $=$ Area of square

$-$ (Area of four quadrants $+$ Area of circle)

$$= 16 - \left(\frac{22}{7} + \frac{22}{7}\right) = 16 - \frac{44}{7} = \frac{112 - 44}{7} = \frac{68}{7}\ \text{cm}^2$$

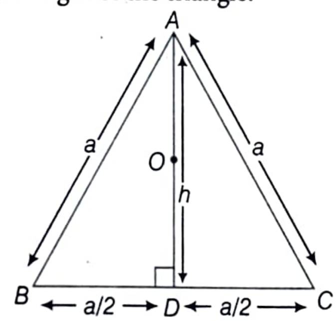
6 In a circular table cover of radius 32 cm, a design is formed leaving an equilateral ΔABC in the middle as shown in the figure. Find the area of the design.



Sol. Given, radius of the circle $= 32$ cm

Let the side of the equilateral ΔABC be a cm.

Let h be the height of the triangle.



We know that in an equilateral triangle, centroid and circumcentre coincide.

$\therefore \quad AO = \dfrac{2}{3}h$ cm

[$\because$ centroid divides the median in the ratio 2 : 1]

which is equal to the radius of circle.

$\therefore \quad \dfrac{2}{3}h = 32 \Rightarrow h = 48$ cm $\quad$ ...(i)

Now, we draw a perpendicular from vertex A to side BC which bisects BC at D.

In right angled ΔADB,

$AB^2 = BD^2 + AD^2$ [by Pythagoras theorem]

$\Rightarrow \quad a^2 = \left(\dfrac{a}{2}\right)^2 + h^2 \Rightarrow h^2 = a^2 - \dfrac{a^2}{4} = \dfrac{3a^2}{4}$

$\Rightarrow \quad (48)^2 = \dfrac{3a^2}{4}$ $\quad$ [from Eq. (i)]

$\Rightarrow \quad a^2 = 3072$

$\Rightarrow \quad a = \sqrt{3072}$ [taking positive square root]

$= 32\sqrt{3}$ cm

$\therefore$ Area of equilateral $\Delta ABC = \dfrac{\sqrt{3}}{4}(a)^2 = \dfrac{\sqrt{3}}{4}(32\sqrt{3})^2$

$$= \frac{\sqrt{3}}{4} \times 3072 = 768\sqrt{3}\ \text{cm}^2$$

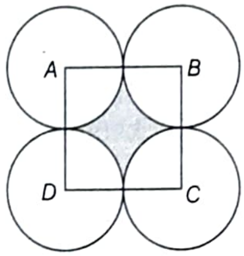
$\therefore$ Area of shaded region

$= $ Area of circle $-$ Area of ΔABC

$$= \pi(32)^2 - 768\sqrt{3} = \frac{22}{7}(1024) - 768\sqrt{3}$$

$$= \left(\frac{22528}{7} - 768\sqrt{3}\right)\text{cm}^2$$

7 In the adjoining figure, *ABCD* is a square of side 14 cm. With centres *A*, *B*, *C* and *D*, four circles are drawn such that each circle touch externally two of the remaining three circles. Find the area of the shaded region.



Sol. Given, side of square = 14 cm

i.e. $AB = BC = CD = DA = 14$ cm

$\therefore$ Radius of circle $= \frac{1}{2}$ (Side of a square) $= \frac{14}{2} = 7$ cm

Area of quadrant of one circle $= \frac{\pi r^2}{4} = \frac{22}{7 \times 4}(7)^2 = \frac{154}{4}$ cm^2

$\therefore$ Area of four quadrants of four circles $= 4\left(\frac{154}{4}\right) = 154$ cm^2

Now, area of square = (Side)$^2 = (14)^2 = 196$ cm^2

Hence, area of shaded region

= Area of square − Area of four quadrants

$= 196 - 154 = 42$ cm^2

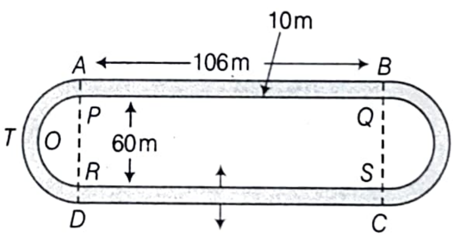
8 Following figure depicts a racing track whose left and right ends are semi-circular. The distance between two inner parallel line segments is 60 m and they are each 106 m long. If the track is 10 m wide, find

(*i*) the distance around the track along its inner edge.

(*ii*) the area of the track.

Sol. Given, length of each parallel lines = 106 m

and width of the track = 10 m



(*i*) Distance around the track along its inner edge

$= PQ + RS + 2 \times$ Circumference of semi-circle *POR*

$$= 106 + 106 + 2\left[\frac{2\pi\left(\frac{60}{2}\right)}{2}\right]$$

$$= 212 + 60\pi = 212 + 60 \times \frac{22}{7} = 212 + \frac{1320}{7}$$

$$= \frac{1484 + 1320}{7} = \frac{2804}{7} \text{ m}$$

(*ii*) Area of the track = Area of outer track − Area of inner track

= [Area of rectangle *ABCD* + 2 × Area of semi-circle *ATD*] − [Area of rectangle*PQRS* + 2 × Area of 4 semi-circle *POR*]

$$= \left[AB \times AD + 2 \times \frac{\pi}{2} \times \left(\frac{AD}{2}\right)^2\right] - \left[PQ \times PR + 2 \times \frac{\pi}{2} \times \left(\frac{PR}{2}\right)^2\right]$$

$$= \left[106 \times (60+20) + \frac{22}{7}\left(\frac{60+20}{2}\right)^2\right] - \left[106 \times 60 + \frac{22}{7}\left(\frac{60}{2}\right)^2\right]$$

$$= \left(106 \times 80 + \frac{22}{7} \times 40 \times 40\right) - \left(106 \times 60 + \frac{22}{7} \times 30 \times 30\right)$$

$$= \left(8480 + \frac{35200}{7}\right) - \left(6360 + \frac{19800}{7}\right)$$

$$= 2120 + \frac{15400}{7} = 2120 + 2200 = 4320 \text{ m}^2$$

Alternate Method

Area of the track = 2 × Area of rectangle *APQB* + 2 × Area of semi circular ring *ATDROPA*

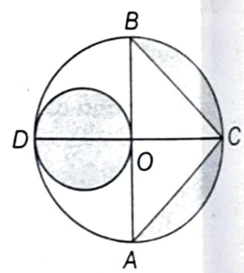
$$= 2 \times 106 \times 10 + 2 \times \frac{\pi}{2}[(40)^2 - (30)^2]$$

$$\left[\because \text{ area of rectangle} = l \times b \text{ and area of semi-circular ring} = \frac{\pi}{2}(R^2 - r^2)\right]$$

$$= 2 \times 106 \times 10 + \frac{22}{7} \times 70 \times 10$$

$$= 2120 + 2200 = 4320 \text{ cm}^2$$

9 In the given figure, *AB* and *CD* are two diameters of a circle (with centre *O*) perpendicular to each other and *OD* is the diameter of the smaller circle. If *OA* = 7 cm, then find the area of the shaded region. **CBSE 2013, 12, 11**



Sol. Given, $OA = 7$ cm

$\therefore$ $OD = 7$ cm [radii of same circle]

$\because$ *OD* is the diameter of smaller circle, then radius $= \frac{7}{2}$ cm

Now, area of smaller circle

$$= \pi r^2 = \pi\left(\frac{7}{2}\right)^2 = \frac{22}{7} \times \frac{49}{4} = \frac{77}{2} \text{ cm}^2$$

Now area of $\Delta ABC = \frac{1}{2} \times AB \times OC$

$= \frac{1}{2} \times 2 \times OA \times OC$ $[\because AB = 2OA]$

$= \frac{1}{2} \times 2 \times 7 \times 7 = 49\text{ cm}^2$ $[\because OA = OC = 7\text{ cm}]$

and area of semi-circle $ABCA = \frac{\pi R^2}{2} = \frac{22}{7 \times 2} \times (7)^2$

$= 77\text{ cm}^2$

$\therefore$ Area of minor segment corresponding to the chords BC and AC = Area of semi-circle – Area of ΔABC

$= 77 - 49 = 28\text{ cm}^2$

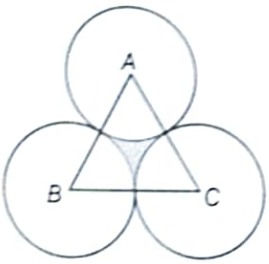
Hence, area of total shaded region

= Area of small circle + Area of minor segment corresponding to the chords BC and AC

$= \frac{77}{2} + 28 = 38.5 + 28 = 66.5\text{ cm}^2$

10 The area of an equilateral ΔABC is 17320.5 cm^2. With each vertex of the triangle as centre, a circle is drawn with radius equal to half the length of the side of the triangle (see the figure). Find the area of the shaded region.

[Take, $\pi = 3.14$ and $\sqrt{3} = 1.73205$]



Sol. Let the side of an equilateral triangle be a.

$\therefore$ Area of an equilateral $\Delta ABC = \frac{\sqrt{3}}{4}(a)^2$

But given, area of equilateral $\Delta ABC = 17320.5\text{ cm}^2$

$\therefore \quad 17320.5 = \frac{1.73205}{4}(a^2)$

$\Rightarrow \quad a^2 = 10000 \times 4 \Rightarrow a = 100 \times 2 = 200\text{ cm}$

[taking positive square root]

Since, ΔABC is an equilateral triangle.

$\therefore \quad \angle A = \angle B = \angle C = 60^\circ$

Here, $AB = a = 200$ cm

$\therefore$ Radius of circle $= \frac{200}{2} = 100$ cm

[$\because$ radius of circle = half the length of the side of the ΔABC]

Area of sector of a circle

$= \frac{\theta}{360^\circ} \times \pi r^2 = \frac{60^\circ}{360^\circ} \times 3.14 \times (100)^2 = 5233.33\text{ cm}^2$

Then, area of three equal sectors

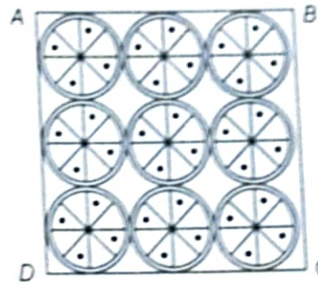
$= 3 \times 5233.33 = 15700\text{ cm}^2$

$\therefore$ Area of required shaded region

= Area of ΔABC – Area of three sectors

$= 17320.5 - 15700 = 1620.5\text{ cm}^2$

11 On a square handkerchief, nine circular designs each of radius 7 cm are made (see the figure). Find the area of the remaining portion of the handkerchief.



Sol. Given, radius of each circle, $r = 7$ cm

$\therefore$ Diameter of circle, $d = 14$ cm

[$\because$ diameter = 2 × radius]

In the given figure, horizontal three circles touch each other.

$\therefore$ Length of a side of square = 3 × Diameter of one circle

$= 3 \times 14 = 42$ cm

Now, area of one circle $= \pi r^2 = \pi (7)^2$

$= \frac{22}{7} \times (7)^2 = 154\text{ cm}^2$

$\therefore$ Area of nine circles $= 9 \times 154 = 1386\text{ cm}^2$

Area of square $ABCD = (\text{Side})^2 = (42)^2 = 1764\text{ cm}^2$

Hence, area of the remaining portion of the handkerchief

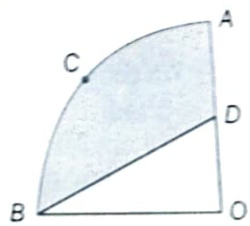
= Area of square – Area of nine circles

$= 1764 - 1386 = 378\text{ cm}^2$

12 In the given figure, $OACB$ is a quadrant of a circle with centre O and radius 3.5 cm. If $OD = 2$ cm, then find the area of the

(i) quadrant $OACB$.

(ii) shaded region.



Sol. Given, radius of quadrant, $r = 3.5$ cm

(i) Area of quadrant $OACB = \frac{1}{4} \times$ Area of the circle

$= \frac{\pi r^2}{4} = \frac{22}{7 \times 4}(3.5)^2 = \frac{22}{7 \times 4}\left(\frac{7}{2}\right)^2 = \frac{22 \times 7}{16} = \frac{77}{8}\text{ cm}^2$

(ii) Now, area of $\Delta BOD = \frac{1}{2} \times OB \times OD = \frac{1}{2} \times 3.5 \times 2$

$= 3.5\text{ cm}^2$

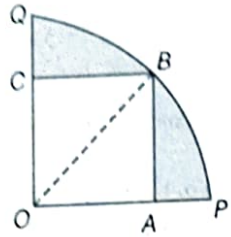
[$\because$ radius $= OB = 3.5$ cm and $OD = 2$ cm, given]

$\therefore$ Area of shaded region

= Area of quadrant $OACB$ – Area of ΔOBD

$= \frac{77}{8} - 3.5 = \frac{77 - 28}{8} = \frac{49}{8}\text{ cm}^2$

13 In the given figure, a square $OABC$ is inscribed in a quadrant $OPBQ$. If $OA = 20$ cm, then find the area of the shaded region.
[Take, $\pi = 3.14$]



Sol. Given, $OABC$ is a square.

$\therefore$ Diagonal of a square $= \sqrt{2} \times \text{Side} = \sqrt{2} \times 20$ cm

[$\because$ side $OA = 20$ cm, given]

Then, radius of circle, $r =$ Diagonal of square $= 20\sqrt{2}$ cm

$\therefore$ Area of quadrant $OPBQO = \dfrac{\pi r^2}{4}$

$$= \frac{3.14 \times (20\sqrt{2})^2}{4} = \frac{3.14 \times 800}{4} = 628 \text{ cm}^2$$

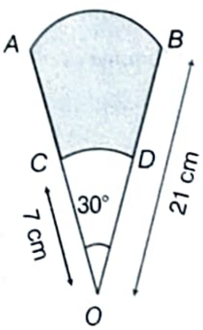
Now, area of square $= (\text{Side})^2 = (20)^2 = 400$ cm^2

Hence, area of shaded region

$= $ Area of quadrant $-$ Area of square

$= 628 - 400 = 228$ cm^2

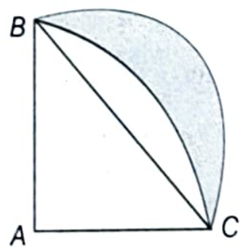
14 AB and CD are respectively arcs of two concentric circles of radii 21 cm and 7 cm and centre O (see the figure). If $\angle AOB = 30°$, then find the area of the shaded region.



Sol. Do same as Q. 2

Ans. $\dfrac{308}{3}$ cm^2

15 In the given figure, ABC is a quadrant of a circle of radius 14 cm and a semi-circle is drawn with BC as diameter. Find the area of the shaded region. **CBSE 2012, 11**



Sol. Given, radius of quadrant,

$$r = 14 \text{cm} = AC = AB$$

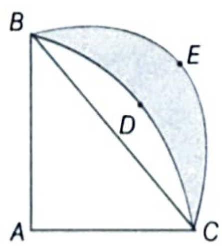
In right angled ΔBAC,

$BC^2 = AB^2 + AC^2$ [by Pythagoras theorem]

$\Rightarrow BC^2 = (14)^2 + (14)^2$

$\Rightarrow BC = 14\sqrt{2}$ cm

[taking positive square root]



Now, area of right angled ΔBAC

$$= \frac{1}{2} \times AC \times AB$$

[$\because$ area of $\Delta = \dfrac{1}{2} \times \text{base} \times \text{height}$]

$$= \frac{1}{2} \times 14 \times 14 = 98 \text{ cm}^2$$

and area of quadrant $ABDC = \dfrac{\pi r^2}{4} = \dfrac{22}{7 \times 4}(14)^2 = 154$ cm^2

Now, area of semi-circle $BCE = \dfrac{\pi r^2}{2}$

$$= \frac{22}{7 \times 2}\left(\frac{14\sqrt{2}}{2}\right)^2 \qquad \left[\because r = \frac{BC}{2}\right]$$

$$= \frac{22 \times 14 \times 14 \times 2}{7 \times 2 \times 4} = 22 \times 7 = 154 \text{ cm}^2$$

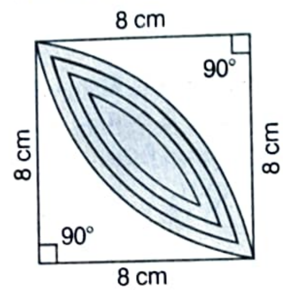
Hence, required area of shaded region

$= $ Area of semi-circle $BCE -$ (Area of quadrant $ABDC$

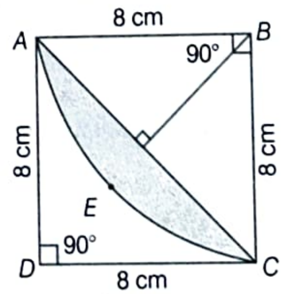
$-$ Area of right angled ΔBAC)

$= 154 - (154 - 98) = 98$ cm^2

16 Calculate the area of the designed region in figure, common between the two quadrants of circles of radius 8 cm each.



Sol. First, we determine the area of the shaded region AEC.



Now, area of sector $BAECB$

$$= \frac{\pi r^2}{4} = \frac{22}{7} \times \frac{(8)^2}{4} = \frac{352}{7} \text{ cm}^2 \quad [\because r = AB = 8 \text{ cm, given}]$$

Now, area of right angled

$$\Delta ABC = \frac{1}{2} \times AB \times BC$$

$$= \frac{1}{2} \times 8 \times 8 = 32 \text{ cm}^2 \qquad [\because BC = 8, \text{ given}]$$

$\therefore$ Area of shaded region AEC

$= $ Area of sector $BAECB -$ Area of right angled ΔABC

$$= \frac{352}{7} - 32 = \frac{352 - 224}{7} = \frac{128}{7} \text{ cm}^2$$

Hence, required shaded region

$= 2 \times$ Area of shaded region AEC

$$= 2 \times \frac{128}{7} = \frac{256}{7} \text{ cm}^2$$