EXERCISE 6.5

1 Sides of triangles are given below. Determine which of them are right angled triangles? In case of a right angled triangle, write the length of its hypotenuse.

(i) 7 cm, 24 cm and 25 cm

(ii) 3 cm, 8 cm and 6 cm

(iii) 50 cm, 80 cm and 100 cm

(iv) 13 cm, 12 cm and 5 cm

> Use Pythagoras theorem, i.e. if the square of the hypotenuse is equal to the sum of the other two sides. Then, the given triangle is a right angled triangle, otherwise not.

Sol. (i) Here, $(7)^2 + (24)^2 = 49 + 576 = 625 = (25)^2$

Therefore, given sides 7 cm, 24 cm and 25 cm make a right angled triangle.

(ii) $(3)^2 + (6)^2 = 9 + 36 = 45 \neq 64$ $[\because 8^2 = 64]$

Therefore, given sides do not make a right angled triangle.

(iii) $(50)^2 + (80)^2 = 2500 + 6400$

$= 8900 \neq 10000$ $[\because (100)^2 = 10000]$

Therefore, given sides do not make a right angled triangle.

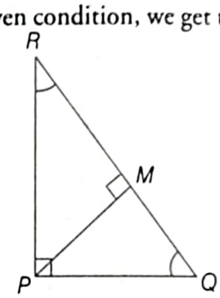
(iv) $(12)^2 + (5)^2 = 144 + 25 = 169 = (13)^2$

Therefore, given sides 13 cm, 12 cm and 5 cm make a right angled triangle.

2 PQR is a triangle right angled at P and M is a point on QR, such that $PM \perp QR$. Show that $PM^2 = QM \times MR$.

> First, show that $\Delta QMP \sim \Delta PMR$ and then use the theorem that the ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides.

Sol. According to given condition, we get the following figure.



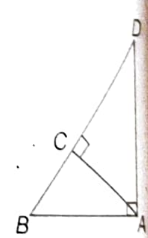
We know that, if a perpendicular is drawn from the vertex of the right angle of a right angled triangle to the hypotenuse, then triangle on both sides of the perpendicular are similar to the whole triangle and to each other.

$\therefore \quad \Delta QMP \sim \Delta PMR$

Now, $\dfrac{QM}{PM} = \dfrac{MP}{MR}$ [$\because$ corresponding sides of similar triangles are proportional]

$\Rightarrow PM^2 = QM \times MR$ Hence proved.

3 In the following figure, ABD is a triangle right angled at A and $AC \perp BD$. Show that



(i) $AB^2 = BC \cdot BD$

(ii) $AC^2 = BC \cdot DC$

(iii) $AD^2 = BD \cdot CD$

Sol. (i) We know that, if a perpendicular is drawn from the vertex of the right angle of a right angled triangle to the hypotenuse, then triangle on both sides of the perpendicular are similar to the whole triangle and to each other.

So, $\Delta ACB \sim \Delta DAB \sim \Delta DCA$ ...(i)

From Eq. (i), we have

$$\Delta ACB \sim \Delta DAB \Rightarrow \frac{AB}{DB} = \frac{CB}{AB}$$

[$\because$ corresponding sides of similar triangles are proportional]

$\Rightarrow \quad AB^2 = BC \cdot BD$ Hence proved.

(*ii*) Do same as Q. 2.

(*iii*) From Eq. (i), we have

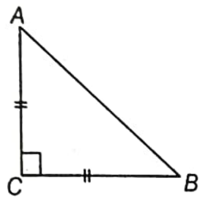
$$\Delta DCA \sim \Delta DAB \Rightarrow \frac{DA}{DB} = \frac{DC}{DA}$$

[$\because$ corresponding sides of similar triangles are proportional]

$\Rightarrow \quad AD^2 = BD \cdot CD$ **Hence proved.**

4 ABC is an isosceles triangle, right angled at C. Prove that $AB^2 = 2AC^2$. **CBSE 2015, 14**

Sol. Given, an isosceles ΔABC, which makes right angle at C and $AC = BC$.



By Pythagoras theorem, we have

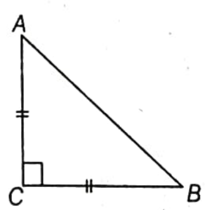
$$AB^2 = AC^2 + BC^2$$

$$= AC^2 + AC^2 \quad [\because BC = AC, \text{given}]$$

$\therefore \quad AB^2 = 2AC^2$ **Hence proved.**

5 ABC is an isosceles triangle with AC = BC. If $AB^2 = 2AC^2$, then prove that ABC is a right angled triangle.

Sol. Given, an isosceles ΔABC with $AC = BC$.



In ΔABC, $\quad AC = BC$ [given] ...(i)

and $\quad AB^2 = 2AC^2$ [given] ...(ii)

Now, $\quad AC^2 + BC^2 = AC^2 + AC^2$ [from Eq. (i)]

$= 2AC^2 = AB^2$ [from Eq. (ii)]

$\Rightarrow \quad AC^2 + BC^2 = AB^2$

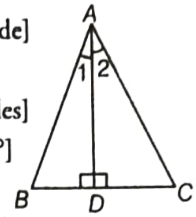
Hence, by converse of Pythagoras theorem, ΔABC is a right angled triangle right angled at C. **Hence proved.**

6 ABC is an equilateral triangle of side 2*a*. Find each of its altitudes.

First, prove that altitude of an equilateral triangle bisects the opposite side and then use Pythagoras theorem in ΔADB or ΔADC, to find altitude.

Sol. Given, an equilateral ΔABC having each side as $2a$. Now, let $AD \perp BC$. Then,

In ΔADB and ΔADC,



$AD = AD$ [common side]

$AB = AC$ [sides of equilateral triangles]

and $\angle ADB = \angle ADC$ [each 90°]

$\therefore \quad \Delta ADB \cong \Delta ADC$ [by RHS congruency rule]

Then, $BD = CD = \frac{1}{2} BC = a$ [by CPCT]

In ΔADB, using Pythagoras theorem, we get

$$AB^2 = AD^2 + BD^2$$

$\Rightarrow \quad (2a)^2 = AD^2 + a^2$

$\Rightarrow \quad 4a^2 = AD^2 + a^2 \Rightarrow AD^2 = 3a^2$

$\therefore \quad AD = \sqrt{3}a$

Hence, the altitude of ΔABC is $\sqrt{3}a$.

7 Prove that the sum of the squares of the sides of a rhombus is equal to the sum of the squares of its diagonals. **CBSE 2015**

In a rhombus, all sides are equal and the diagonals bisect each other perpendicularly. Use this result and then apply Pythagoras theorem.

Sol. Suppose $ABCD$ is a rhombus in which

$$AB = BC = CD = DA = a \quad \text{[say]}$$

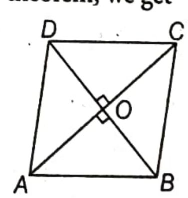
Here, diagonals AC and BD are right angle bisectors of each other at O.

In ΔAOB, $\angle AOB = 90°$

$OA = \frac{1}{2} AC$ [$\because OA = OC$] ...(i)

and $\quad OB = \frac{1}{2} BD$ [$\because OB = OD$] ...(ii)

Using Pythagoras theorem, we get



$$OA^2 + OB^2 = AB^2$$

$\Rightarrow \left(\frac{1}{2} AC\right)^2 + \left(\frac{1}{2} BD\right)^2 = AB^2$ [from Eqs. (i) and (ii)]

$\Rightarrow \quad AC^2 + BD^2 = 4AB^2$

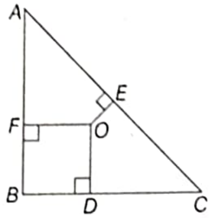
$\Rightarrow AB^2 + BC^2 + CD^2 + DA^2 = AC^2 + BD^2$

[$\because AB = BC = CD = DA$]

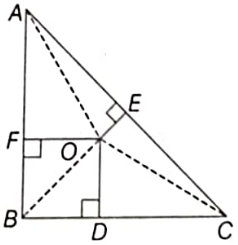
Hence proved.

8 In the given figure, O is a point in the interior of a ΔABC, $OD \perp BC$, $OE \perp AC$ and $OF \perp AB$.
Show that
(i) $OA^2 + OB^2 + OC^2 - OD^2 - OE^2 - OF^2 = AF^2 + BD^2 + CE^2$

(ii) $AF^2 + BD^2 + CE^2 = AE^2 + CD^2 + BF^2$



Sol. In ΔABC, from point O, join lines OA, OB and OC.



(*i*) In right angled ΔOFA, using Pythagoras theorem, we get

$$OA^2 = OF^2 + AF^2$$

$$\Rightarrow OA^2 - OF^2 = AF^2 \quad \text{...(i)}$$

Similarly, in ΔODB,

$$OB^2 - OD^2 = BD^2 \quad \text{...(ii)}$$

and in ΔOEC,

$$OC^2 - OE^2 = CE^2 \quad \text{...(iii)}$$

On adding Eqs. (i), (ii) and (iii), we get

$$OA^2 + OB^2 + OC^2 - OD^2 - OE^2 - OF^2 = AF^2 + BD^2 + CE^2$$

(*ii*) From part (i),

$$OA^2 + OB^2 + OC^2 - OD^2 - OE^2 - OF^2 = AF^2 + BD^2 + CE^2 \quad \text{...(iv)}$$

Similarly, we can prove that

$$OA^2 + OB^2 + OC^2 - OD^2 - OE^2 - OF^2 = BF^2 + CD^2 + AE^2 \quad \text{...(v)}$$

From Eqs. (iv) and (v),

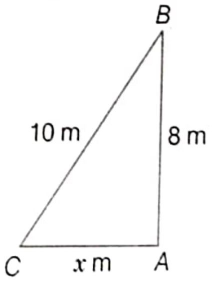
$$AF^2 + BD^2 + CE^2 = AE^2 + CD^2 + BF^2$$

Hence proved.

9 A ladder 10 m long reaches a window 8 m above the ground. Find the distance of the foot of the ladder from base of the wall.

Sol. Let AB be the position of a window from the ground and BC be the ladder, then height of window, $AB = 8$ m and length of ladder, $BC = 10$ m

Let $AC = x$ m be the distance of the foot of the ladder from the base of the wall.



In ΔBAC, using Pythagoras theorem, we get

$$AC^2 + AB^2 = BC^2$$

$$\Rightarrow x^2 + (8)^2 = (10)^2$$

$$[\because AB = 8\,\text{m and } BC = 10\,\text{m}]$$

$$\Rightarrow x^2 = 100 - 64 = 36$$

$$\Rightarrow x = \sqrt{36} \quad \text{[taking positive square root]}$$

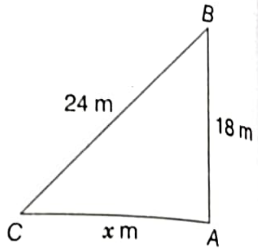
$$\therefore x = 6, \quad \text{i.e.} \quad AC = 6\,\text{m}$$

Hence, the distance of the foot of the ladder from base of the wall is 6 m.

10 A guy wire attached to a vertical pole of height 18 m is 24 m long and has a stake attached to the other end. How far from the base of the pole should the stake be driven so that the wire will be taut?

Here, the vertical pole makes a right angle with ground, so use Pythagoras theorem to find the required value.

Sol. Let AB be the vertical pole of height 18 m and guy wire BC of length 24 m. Let $AC = x$ m be the distance of the stake from the base of the pole.



In ΔBAC, using Pythagoras theorem, we get

$$AC^2 + AB^2 = BC^2$$

$$\Rightarrow x^2 + (18)^2 = (24)^2$$

$$[\because AC = x\,\text{m}, AB = 18\,\text{m and } BC = 24\,\text{m}]$$

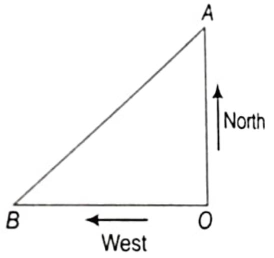
$$\Rightarrow x^2 = (24)^2 - (18)^2 = 576 - 324 = 252$$

$$\therefore x = \sqrt{252} = 6\sqrt{7} \quad \text{[taking positive square root]}$$

Hence, the distance of the stake from the base of pole is $6\sqrt{7}$ m.

11 An aeroplane leaves an airport and flies due North at a speed of 1000 km/h. At the same time, another aeroplane leaves the same airport and flies due West at a speed of 1200 km/h. How far apart will be the two planes after $1\frac{1}{2}$ h?

Sol. Let O be the position of airport.



In $1\frac{1}{2}$ h, the distance travelled by aeroplance when it flies due North at a speed of 1000 km/h,

$$OA = 1000 \times \frac{3}{2} = 1500\,\text{km} \quad [\because \text{distance} = \text{speed} \times \text{time}]$$

In $1\frac{1}{2}$ h, the distance travelled by aeroplane when it flies due West at a speed of 1200 km/h, $OB = 1200 \times \frac{3}{2} = 1800$ km

$$[\because \text{distance} = \text{speed} \times \text{time}]$$

Since, North and West directions are perpendicular to each other.

So, using Pythagoras theorem in right ΔAOB, we get

$$AB^2 = OB^2 + OA^2 \Rightarrow AB^2 = (1800)^2 + (1500)^2$$

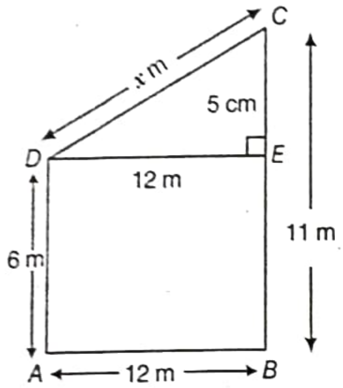
$$\Rightarrow AB^2 = 3240000 + 2250000 = 5490000$$

$$\therefore AB = 300\sqrt{61}\,\text{km} \quad \text{[taking positive square root]}$$

Hence, the distance between two planes after $1\frac{1}{2}$ h is $300\sqrt{61}$ km.

12 Two poles of heights 6 m and 11 m stand on a plane ground. If the distance between the feet of the poles is 12 m, find the distance between their tops.

Sol. Let BC and AD be the two poles of height 11 m and 6 m, respectively. Again, let x m be the distance between tops of the poles.



Then, $CE = BC - AD = 11 - 6 = 5\text{ m}$ $[\because AD = BE]$

Also, $AB = 12\text{ m}$

Again, let distance between the tops of two poles be

$$DC = x\text{ m}$$

In ΔDEC, using Pythagoras theorem, we get

$$DC^2 = DE^2 + CE^2$$

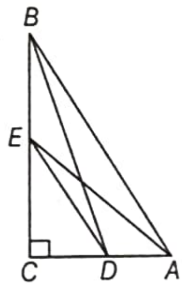
$$\Rightarrow \quad x^2 = 12^2 + 5^2 = 144 + 25 = 169$$

$$\therefore \quad x = \sqrt{169} = 13 \quad \text{[taking positive square root]}$$

Hence, the distance between the tops of the poles is 13 m.

13 D and E are points on the sides CA and CB respectively of ΔABC right angled at C. Prove that $AE^2 + BD^2 = AB^2 + DE^2$.

Sol. Consider a right angled ΔACB, right angled at C. D and E are two points on sides CA and CB. Join ED, BD and EA.



In right angled ΔACE, using Pythagoras theorem, we get

$$AE^2 = CA^2 + CE^2 \quad \text{...(i)}$$

In right angled ΔBCD, using Pythagoras theorem, we get

$$BD^2 = BC^2 + CD^2 \quad \text{...(ii)}$$

In ΔECD, using Pythagoras theorem, we get

$$CD^2 + CE^2 = DE^2 \quad \text{...(iii)}$$

and in ΔACB, using Pythagoras theorem, we get

$$CA^2 + CB^2 = BA^2 \quad \text{...(iv)}$$

On adding Eqs. (i) and (ii), we get

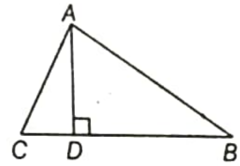
$$AE^2 + BD^2 = (CA^2 + CE^2) + (BC^2 + CD^2)$$

$$= (BC^2 + CA^2) + (CD^2 + CE^2)$$

$$= BA^2 + DE^2 \quad \text{[from Eqs. (iii) and (iv)]}$$

$$\Rightarrow \quad AE^2 + BD^2 = AB^2 + DE^2$$ **Hence proved.**

14 The perpendicular from A on side BC of a ΔABC intersects BC at D such that $DB = 3CD$ (see figure). Prove that



$$2AB^2 = 2AC^2 + BC^2.$$ **CBSE 2012**

Sol. Given, $DB = 3CD$ ...(i)

Now, $BC = BD + CD$

$= 3CD + CD = 4CD$ [from Eq. (i)]

$$\Rightarrow \quad CD = \frac{1}{4}BC \quad \text{...(ii)}$$

On substituting $CD = \frac{1}{4}BC$ in Eq. (i), we get

$$DB = \frac{3}{4}BC \quad \text{...(iii)}$$

In ΔADB, using Pythagoras theorem, we get

$$AB^2 = DB^2 + AD^2 \quad \text{...(iv)}$$

In ΔADC, using Pythagoras theorem, we get

$$AC^2 = CD^2 + AD^2 \quad \text{...(v)}$$

On subtracting Eq. (v) from Eq. (iv), we get

$$AB^2 - AC^2 = DB^2 - CD^2 = \left(\frac{3}{4}BC\right)^2 - \left(\frac{1}{4}BC\right)^2$$

[from Eqs. (ii) and (iii)]

$$\Rightarrow \quad AB^2 - AC^2 = \frac{9}{16}BC^2 - \frac{1}{16}BC^2 = \frac{1}{2}BC^2$$

$$\Rightarrow \quad 2AB^2 - 2AC^2 = BC^2$$

$$\Rightarrow \quad 2AB^2 = 2AC^2 + BC^2$$ **Hence proved.**

15 In an equilateral ΔABC, D is a point on side BC, such that $BD = 1/3\, BC$. Prove that $9AD^2 = 7AB^2$.

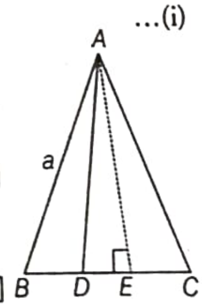
CBSE 2014

Sol. Draw an equilateral ΔABC.

Given, D is a point on side BC, such that

$$BD = 1/3\ BC \quad \text{...(i)}$$

Draw a line $AE \perp BC$.



Since, ΔABC is an equilateral triangle.

$\therefore \quad AB = BC = CA = a$ [say]

and $BD = \frac{1}{3}BC = \frac{1}{3}a$ [from Eq. (i)]

Now, $CD = BC - BD$

$= BC - \frac{1}{3}BC$ [from Eq. (i)]

$= \frac{2}{3}BC = \frac{2}{3}a$

$\because \quad AE \perp BC \quad \therefore \quad BE = EC = \frac{1}{2}a$

[since, in an equilateral triangle, altitude AE is perpendicular bisector of BC]

Now, $DE = BE - BD$

$$= \frac{1}{2}a - \frac{1}{3}a = \frac{1}{6}a$$

In ΔAED, using Pythagoras theorem, we get

$$AD^2 = AE^2 + DE^2 = AB^2 - BE^2 + DE^2$$

[since, in ΔAEB, $AE^2 = AB^2 - BE^2$]

$$= a^2 - \left(\frac{1}{2}a\right)^2 + \left(\frac{1}{6}a\right)^2 = a^2 - \frac{1}{4}a^2 + \frac{1}{36}a^2$$

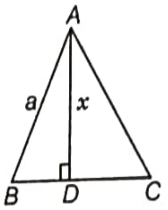
$$\Rightarrow \quad AD^2 = \frac{(36 - 9 + 1)\,a^2}{36} = \frac{28}{36}a^2 = \frac{7}{9}AB^2$$

$$\Rightarrow \quad 9AD^2 = 7AB^2$$

Hence proved.

16 In an equilateral triangle, prove that three times the square of one side is equal to four times the square of one of its altitudes.

Sol. Let ΔABC be an equilateral triangle of side a.



Draw $AD \perp BC$. Again, let $AD = x$ is an altitude.

Now, $BD = CD$

[since, in an equilateral triangle, altitude AD is perpendicular bisector of BC]

$$= \frac{1}{2}BC = \frac{1}{2}a$$

[$\because$ each side of an equilateral triangle is a]

In right angled ΔADB, using Pythagoras theorem, we get

$$AB^2 = AD^2 + BD^2$$

$$\Rightarrow \quad a^2 = x^2 + \left(\frac{1}{2}a\right)^2$$

$$\Rightarrow \quad a^2 = x^2 + \frac{1}{4}a^2 \Rightarrow 4a^2 = 4x^2 + a^2$$

$$\therefore \quad 3a^2 = 4x^2$$

Thus, we have three times the square of one side (a) is equal to four times the square of one of its altitudes.

Hence proved.

Tick the Correct Answer and Justify

17 In ΔABC, $AB = 6\sqrt{3}$ cm, $AC = 12$ cm and $BC = 6$ cm. Then, $\angle B$ is

(a) 120° (b) 60°

(c) 90° (d) 45°

Sol. (c) Given, $AB = 6\sqrt{3}$ cm, $AC = 12$ cm and $BC = 6$ cm

Now, $AB^2 + BC^2 = (6\sqrt{3})^2 + 6^2 = 36 \times 3 + 36 = 108 + 36$

$$= 144 = (12)^2 = (AC)^2$$

Thus, ΔABC is right angled at B.

[by the converse of Pythagoras theorem]

$$\therefore \quad \angle B = 90°$$