EXERCISE 6.4

1 Let $\Delta ABC \sim \Delta DEF$ and their areas be respectively $64\ cm^2$ and $121\ cm^2$.

If $EF = 15.4$ cm, then find BC.

Use the theorem (or property) that the ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides.

Sol. Given, $\Delta ABC \sim \Delta DEF$

$\therefore \quad \dfrac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta DEF)} = \dfrac{BC^2}{EF^2}$

[using property of area of similar triangles]

$\Rightarrow \quad \dfrac{64}{121} = \dfrac{BC^2}{EF^2} \Rightarrow \left(\dfrac{BC}{EF}\right)^2 = \left(\dfrac{8}{11}\right)^2$

$\Rightarrow \quad \dfrac{BC}{EF} = \dfrac{8}{11}$ [taking positive square root on both sides]

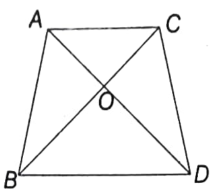
$\Rightarrow \quad BC = \dfrac{8}{11} \times EF$

$\therefore \quad BC = \dfrac{8}{11} \times 15.4 = 11.2\text{ cm}$ [$\because EF = 15.4$ cm, given]

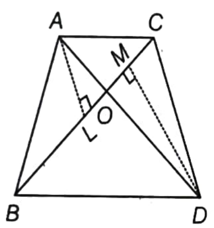
2 Diagonals of a trapezium $ABCD$ with $AB \parallel DC$ intersect each other at the point O. If $AB = 2CD$, then find the ratio of the areas of ΔAOB and ΔCOD. **CBSE 2011**

Sol. Solve as Example 3 of topic 3 **Ans.** 4 : 1

3 In the given figure, ABC and DBC are two triangles on the same base BC. If AD intersects BC at O, then show that $\dfrac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta DBC)} = \dfrac{AO}{DO}$.



Sol. Draw $AL \perp BC$ and $DM \perp BC$. [see figure]



In ΔOLA and ΔOMD,

$\angle ALO = \angle DMO$ [each 90°]

$\angle AOL = \angle DOM$ [vertically opposite angles]

$\therefore \quad \Delta OLA \sim \Delta OMD$ [by AA similarity criterion]

$\Rightarrow \quad \dfrac{AL}{DM} = \dfrac{AO}{DO}$ ...(i)

Now, $\dfrac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta DBC)} = \dfrac{\frac{1}{2} \times BC \times AL}{\frac{1}{2} \times BC \times DM}$

[$\because$ area of triangle = = $\frac{1}{2}$ × base × height]

$= \dfrac{AL}{DM} = \dfrac{AO}{DO}$ [from Eq. (i)]

Hence proved.

4 If the areas of two similar triangles are equal then prove that they are congruent. **CBSE 201**

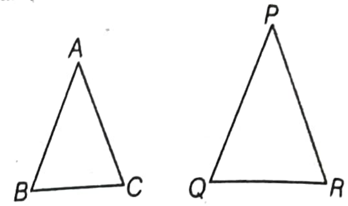
Use the theorem that the ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides, then prove that they are congruent.

Sol. Let the two triangles be ABC and PQR.

Then, according to the question,

$\Delta ABC \sim \Delta PQR$

and $\quad \text{ar}(\Delta ABC) = \text{ar}(\Delta PQR)$ [given]



i.e. $\quad \dfrac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta PQR)} = 1$

$\Rightarrow \quad \dfrac{AB^2}{PQ^2} = \dfrac{BC^2}{QR^2} = \dfrac{CA^2}{PR^2} = 1$

[using theorem of area of similar triangles]

$\Rightarrow \quad AB = PQ,\ BC = QR$ and $CA = PR$

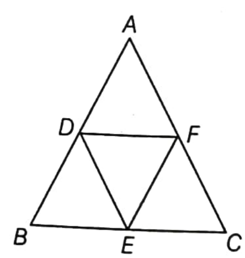
$\therefore \quad \Delta ABC \cong \Delta PQR$ [by SSS congruence rule]

Hence proved.

5 D, E and F are respectively the mid-points of sides AB, BC and CA of ΔABC. Find the ratio of the areas of ΔDEF and ΔABC. **CBSE 2012, 11, 10**

Use the mid-point theorem, which states that 'the line segment joining the mid-points of two sides of a triangle is parallel to the third side and its length is equal to the half of the length of the third side', to find the relation between sides of ΔABC and ΔDEF.

Sol. Draw a ΔABC and D, E and F are the mid-points of sides AB, BC and CA, respectively. Join the points D, E and F.



By mid-point theorem,

$DF = \dfrac{1}{2} BC$

$DE = \dfrac{1}{2} CA$ and $EF = \dfrac{1}{2} AB$ ...(i)

In ΔDEF and ΔCAB,

$$\frac{DF}{BC} = \frac{DE}{CA} = \frac{EF}{AB} = \frac{1}{2} \quad \text{[from Eq. (i)]}$$

$$\Delta DEF \sim \Delta CAB \quad \text{[by SSS similarity criterion]}$$

$\therefore$ Now, $$\frac{ar(\Delta DEF)}{ar(\Delta CAB)} = \frac{DE^2}{CA^2}$$

[using theorem of area of similar triangles]

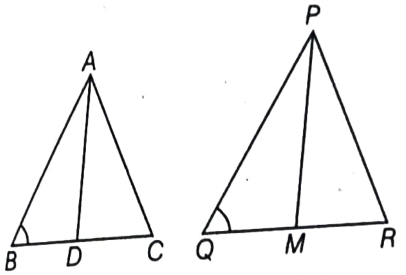
$$= \frac{\left(\frac{1}{2}CA\right)^2}{CA^2} = \frac{1}{4} \quad \text{[from Eq. (i)]}$$

$\therefore$ $$\frac{ar(\Delta DEF)}{ar(\Delta ABC)} = \frac{1}{4} \quad [\because ar(\Delta CAB) = ar(\Delta ABC)]$$

Hence, the required ratio is 1 : 4.

6 Prove that the ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding medians.

Sol. Let ΔABC and ΔPQR be two similar triangles as shown below.



Let AD be a median of ΔABC and PM is a median of ΔPQR. Then, D is the mid-point of BC and M is the mid-point of QR.

Also, we have $\Delta ABC \sim \Delta PQR$ ...(i)

$\Rightarrow$ $\angle B = \angle Q$

[since, corresponding angles are equal]

Also, $$\frac{AB}{PQ} = \frac{BC}{QR}$$

[since, ratio of corresponding sides are equal]

$\Rightarrow$ $$\frac{AB}{PQ} = \frac{2BD}{2QM}$$

[since, D is the mid-point of BC and M is the mid-point of QR]

$\Rightarrow$ $$\frac{AB}{PQ} = \frac{BD}{QM} \quad \text{...(ii)}$$

In ΔABD and ΔPQM,

$$\angle ABD = \angle PQM \quad \text{[from Eq. (i)]}$$

and $$\frac{AB}{PQ} = \frac{BD}{QM} \quad \text{[from Eq. (ii)]}$$

$\therefore$ $\Delta ABD \sim \Delta PQM$ [by SAS similarity criterion]

$\Rightarrow$ $$\frac{AB}{PQ} = \frac{AD}{PM}$$

[since, ratio of corresponding sides are equal] ...(iii)

Now, $$\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{AB^2}{PQ^2}$$

[using theorem of area of similar triangles]

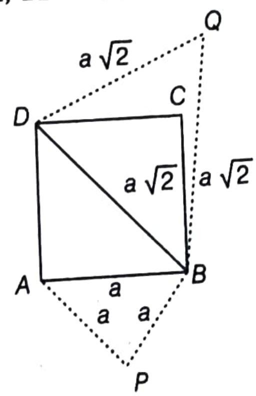
$\therefore$ $$\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{AD^2}{PM^2} \quad \text{[from Eq. (iii)]}$$

Hence proved.

7 Prove that the area of an equilateral triangle described on one side of a square is equal to half the area of an equilateral triangle described on one of its diagonals. **CBSE 2010**

Sol. Let $ABCD$ be a square having sides of length a.

Then, diagonal, $BD = a\sqrt{2}$ [By pythagoras theorem]



Now, construct two equilateral triangles.

$\therefore$ $\Delta PAB \sim \Delta QBD$

[since, equilateral triangles are similar]

$\Rightarrow$ $$\frac{ar(\Delta PAB)}{ar(\Delta QBD)} = \frac{AB^2}{BD^2} = \frac{a^2}{(a\sqrt{2})^2} = \frac{1}{2}$$

[using theorem of area of similar triangles]

$\Rightarrow$ $$ar(\Delta PAB) = \frac{1}{2} ar(\Delta QBD)$$ **Hence proved.**

Tick the Correct Answer and Justify

8 ABC and BDE are two equilateral triangles, such that D is the mid-point of BC. Ratio of the areas of ΔABC and ΔBDE is

(a) 2 : 1 (b) 1 : 2

(c) 4 : 1 (d) 1 : 4

Sol. (c) In ΔABC,

$$AB = BC = CA = a \quad \text{[say]}$$

[since, ΔABC is an equilateral triangle]

Also, $BD = \frac{1}{2}a$ [since, D is the mid-point of BC]

Now, $\Delta ABC \sim \Delta BDE$

[since, both the triangles are equilateral triangles]

$$\therefore \quad \frac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta BDE)} = \frac{AB^2}{BD^2} = \frac{a^2}{\left(\frac{1}{2}a\right)^2} = \frac{4}{1}$$

[by theorem of area of similar triangles]

Hence, the required ratio is 4 : 1.

9 Sides of two similar triangles are in the ratio 4 : 9. Areas of these triangles are in the ratio

(a) 2 : 3 (b) 4 : 9

(c) 81 : 16 (d) 16 : 81

Sol. (d) We know that, areas of two similar triangles are in the ratio of the squares of their corresponding sides.

Here, the sides are in ratio 4 : 9.

$\therefore$ Ratio of areas of these triangles $= (4)^2 : (9)^2 = 16 : 81$